1. Let C be the curve that runs once counterclockwise around the boundary of the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$. Find $\oint_C (x^2+y)\,dx + (2xy^2 - xy)\,dy$.

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2. For the given C and $\mathbf{F}$, find the counterclockwise circulation $\oint_C \mathbf{F}\cdot\mathbf{T}\,ds$ and the outward normal flux $\oint_C \mathbf{F}\cdot\mathbf{N}\,ds$.

   (a) $\mathbf{F} = \langle xy^2,\, y^3\rangle$, C is the boundary of the rectangle with vertices $(0,0)$, $(2,0)$, $(2,3)$, $(0,3)$.

   (b) $\mathbf{F} = \langle 6xy,\, 0\rangle$, C is the boundary of the triangle with vertices $(0,0)$, $(1,0)$, and $(0,2)$.

   (c) $\mathbf{F} = \langle 2x+3y,\, 4x+5y\rangle$, C is the unit circle.

   (d) $\mathbf{F} = \langle x^3 - y^3,\, x^3+y^3\rangle$, C is the boundary of the quarter-disc $x^2+y^2 \le 16$ in the first quadrant.

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3. Let $C = C_1 \cup C_2 \cup C_3$, where C_1 is the line segment from $(-2,0)$ to $(0,0)$, C_2 is the line segment from $(0,0)$ to $(\sqrt{2},\sqrt{2})$, and C_3 is the shorter arc of the circle $x^2+y^2=4$ from $(\sqrt{2},\sqrt{2})$ to $(-2,0)$.

   (a) Find the outward flux of $\mathbf{F} = \langle 2x^3y^2 + y^3,\, x^2 - 2x^2y^3\rangle$ around C.

   (b) Find the counterclockwise circulation of $\mathbf{F} = \langle x^2 - y^3,\, x^3 + y^4\rangle$ around C.

   (c) Find the work done by $\mathbf{F} = \langle -2y, 2x\rangle$ on a particle that travels once around C.

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4. A particle moves through the force field $\mathbf{F}(x,y) = \left\langle 9y - e^{x^2} + 11,\, 4x + \sin\sqrt{y}\right\rangle$ N where x and y are measured in meters.

   (a) Is the vector field $\mathbf{F}$ conservative? If so find a potential function and if not explain why not.

   (b) Calculate the work done by $\mathbf{F}$ if the particle starts at $(0,0)$, moves along a straight line to $(0,4)$, then moves counterclockwise along the circle $x^2+y^2=16$ to $(-4,0)$, then finally moves in a straight line back to $(0,0)$.

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5. Let $\mathbf{F}(x,y) = \langle 1-y,\, \cos(y^2)+2x\rangle$ and let C be the curve starting at $(0,0)$, traveling along a straight line to $(0,3)$, then along a straight line to $(4,3)$, and then along a straight line to $(4,0)$. Find $\int_C \mathbf{F}\cdot d\mathbf{r}$.

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6. Let $\mathbf{F} = \langle 3x^2y, xy^2, 8xy\rangle$.

   (a) Parametrize the portion of the plane $z=0$ with $0 \le x \le 1$ and $0 \le y \le 1$ with downward orientation, and then calculate the flux of $\mathbf{F}$ across this portion of the plane.

   (b) Use the divergence theorem to calculate the flux of $\mathbf{F}$ outward across the surface of the cube whose faces are made up of portions of the six planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$.

   (c) Calculate the flux of $\mathbf{F}$ outward through the surface S made up of the portions of the five planes $x=0$, $x=1$, $y=0$, $y=1$, $z=1$, with $0 \le x,y,z \le 1$.

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7. Use the divergence theorem to calculate the outward flux of $\mathbf{F}(x,y,z) = xz\mathbf{i} + (x^2+yz)\mathbf{j} + z^2\mathbf{k}$ across the surface of the solid region above $z=0$, below $z=x^2+y^2$, and inside $x^2+y^2=1$.

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8. Use Stokes's theorem to calculate the circulation of the vector field $\mathbf{F} = \langle \sin(x^2) - y^2, z, y - e^z\rangle$ around the counterclockwise boundary of the portion of the surface $z = xy$ that lies above the plane region with $0 \le x \le 1$ and $0 \le y \le 2$.

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9. Solve the following problems:

(a) Determine the flux of $\mathbf{F} = \langle xy^2z^2,\ x^2z^2,\ -xy^2 \rangle$ outward through the surface S made up of the portions of the six planes $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1$, with $0 \le x, y, z \le 1$.

(b) Calculate the flux of $\mathbf{F} = \langle x^3z,\ y^3z,\ 0 \rangle$ outward through the boundary of the solid region with $x^2+y^2 \le 4$ and $1 \le z \le 3$.

(c) Find the flux of $\mathbf{F} = \langle xy^2,\ yz^2,\ x^2z \rangle$ through the surface of the unit sphere with outward orientation.

(d) Find the flux of the curl $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, d\sigma$, where $\mathbf{F}(x, y, z) = \langle -2y \cos(z),\ 2x,\ xe^y \rangle$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = 9$ with outward orientation.

(e) Compute the outward flux $\iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma$ where S is the surface of the "ice cream cone" (the portion of the cone $z = \sqrt{x^2 + y^2}$ that lies inside the sphere $x^2 + y^2 + z^2 = 1$ along with that portion of the sphere that lies above the cone), and $\mathbf{F}(x, y, z) = \langle x + 2y^2,\ 5y - 3xz,\ y^2 + 6z \rangle$.

(f) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = \langle x - y^2,\ y - z,\ z^2 + y \rangle$ and C is the rectangle with vertices $(0, 0, 4)$, $(2, 0, 0)$, $(2, 1, 0)$, and $(0, 1, 3)$ lying in the plane $x + 2y + z = 4$, oriented in the counterclockwise direction when viewed from above.

(g) Find the circulation of the vector field $\mathbf{F} = \langle 2xy, x^2, y \rangle$ around the counterclockwise boundary of the portion of the surface $z = x^2y$ that lies above the plane region with $0 \le x \le 1$ and $0 \le y \le x$.

(h) Calculate the flux of $\mathbf{F} = \left\langle xy^2 + e^z,\ x^2y + e^{2z},\ \sqrt{x^2 + y^2} \right\rangle$ through the portion of the surface $z = 1 - x^2 - y^2$ that lies above the xy-plane, with upward orientation.

(i) Evaluate the flux of $\mathbf{F} = \langle x^3 + 2xz^2,\ x^2y + 2yz^2,\ 4y^2z \rangle$ outward through the upper half of the sphere $x^2 + y^2 + z^2 = 5$.
