1. By Green's theorem, $\oint_C P\,dx + Q\,dy = \iint_R (Q_x - P_y)\,dy\,dx = \int_0^1 \int_0^1 (2y^2 - y - 1)\,dy\,dx = \boxed{-5/6}$.

2. By Green, circulation $\oint_C P\,dx + Q\,dy = \iint_R (Q_x - P_y)\,dA$ and flux $\oint_C -Q\,dx + P\,dy = \iint_R (P_x + Q_y)\,dA$.
   (a) Region is $0 \le x \le 2$, $0 \le y \le 3$. Circ is $\int_0^2 \int_0^3 (-2xy)\,dy\,dx = \boxed{-18}$, flux is $\int_0^2 \int_0^3 4y^2\,dy\,dx = \boxed{72}$.
   (b) Region is $0 \le x \le 1$, $0 \le y \le 2-2x$. Circ is $\int_0^1 \int_0^{2-2x} (-6x)\,dy\,dx = \boxed{-2}$, flux is $\int_0^1 \int_0^{2-2x} 6y\,dy\,dx = \boxed{4}$.
   (c) Region is $0 \le r \le 1$, $0 \le \theta \le 2\pi$. Circ is $\int_0^{2\pi} \int_0^1 1 \cdot r\,dr\,d\theta = \boxed{\pi}$, flux is $\int_0^{2\pi} \int_0^1 7 \cdot r\,dr\,d\theta = \boxed{7\pi}$.
   (d) Region is $0 \le r \le 4$, $0 \le \theta \le \pi/2$. Circ is $\int_0^{\pi/2} \int_0^4 3r^2 \cdot r\,dr\,d\theta = \boxed{96\pi}$, flux is $\int_0^{\pi/2} \int_0^4 3r^2 \cdot r\,dr\,d\theta = \boxed{96\pi}$.

3. Note that C is the counterclockwise boundary of the polar region $0 \le r \le 2$ and $\pi/4 \le \theta \le \pi$.
   (a) By Green, flux of $\mathbf{F} = \langle P, Q\rangle$ is $\iint_R (P_x + Q_y)\,dA = \int_{\pi/4}^{\pi} \int_0^2 0 \cdot r\,dr\,d\theta = \boxed{0}$.
   (b) By Green, circulation of $\mathbf{F} = \langle P, Q\rangle$ is $\iint_R (Q_x - P_y)\,dA = \int_{\pi/4}^{\pi} \int_0^2 3r^2 \cdot r\,dr\,d\theta = \boxed{9\pi}$.
   (c) We can use the tangential form of Green's theorem for the work integral, since it is the same as the circulation integral. So the work is $\iint_R (Q_x - P_y)\,dA = \int_{\pi/4}^{\pi} \int_0^2 4 \cdot r\,dr\,d\theta = \boxed{6\pi}$.

4. (a) We compute $\text{curl}(\mathbf{F}) = \langle 0, 0, -5\rangle$. Since this is nonzero, $\mathbf{F}$ is $\boxed{\text{not conservative}}$.
   (b) We can use Green's theorem since this path is the counterclockwise boundary of the polar region $0 \le r \le 4$ and $\pi/2 \le \theta \le \pi$. By Green, the work is $\iint_R (Q_x - P_y)\,dA = \int_{\pi/2}^{\pi} \int_0^4 -5 \cdot r\,dr\,d\theta = \boxed{-20\pi \text{ J}}$.

5. This path is not closed because it is missing the segment from $(4, 0)$ to $(0, 0)$: that segment is parametrized by $\mathbf{r}(t) = \langle 4 - 4t, 0\rangle$ for $0 \le t \le 1$ so the line integral on that segment is $\int_C (1 - y)\,dx + (\cos(y^2) + 2x)\,dy = \int_0^1 1 \cdot (-4\,dt) + (9 - 8t) \cdot (0\,dt) = -2$. If we add that segment back in, we could then use Green's theorem to evaluate the integral along the full path as $\iint_R (Q_x - P_y)\,dA = \int_0^4 \int_0^3 (3)\,dy\,dx = 36$. Therefore, the integral on the three requested pieces is equal to the difference $36 - (-2) = \boxed{38}$.

6. (a) We have a parametrization $\mathbf{r}(s, t) = \langle s, t, 0\rangle$ for $0 \le s \le 1$, $0 \le t \le 1$. Then $\mathbf{n} = (d\mathbf{r}/ds) \times (d\mathbf{r}/dt) = \langle 1, 0, 0\rangle \times \langle 0, 1, 0\rangle = \langle 0, 0, 1\rangle$, but this has the wrong orientation since it must point downward. Then $\mathbf{F} \cdot (-\mathbf{n}) = -8st$, and so the surface integral is $\int_0^1 \int_0^1 -8st\,dt\,ds = \boxed{-2}$.
   (b) The solid is $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$ and also $\text{div}(\mathbf{F}) = 6xy + 2xy + 0 = 8xy$. Thus by the divergence theorem, the flux through the solid is $\iiint_D \text{div}(\mathbf{F})\,dV = \int_0^1 \int_0^1 \int_0^1 8xy\,dz\,dy\,dx = \boxed{2}$.
   (c) The surface is not closed, but we can close it and then subtract the flux through the missing face $z = 0$ with $0 \le x, y \le 1$, which was analyzed in (a). By (b) the total flux is 2, and the flux across the missing face is -2 by (a), so the flux across the remaining five planes is $2 - (-2) = \boxed{4}$.

7. In cylindrical the solid is $0 \le \theta \le 2\pi$, $0 \le r \le 1$, $0 \le z \le r^2$, while $\text{div}(\mathbf{F}) = z + z + 2z = 4z$. Thus by the divergence theorem, the flux is $\iiint_D \text{div}(\mathbf{F})\,dV = \int_0^{2\pi} \int_0^1 \int_0^{r^2} 4z \cdot r\,dz\,dr\,d\theta = \boxed{4\pi/5}$.

8. Parametrize the surface as $\mathbf{r}(s, t) = \langle s, t, st\rangle$ for $0 \le s \le 1$, $0 \le t \le 2$. Then $\nabla \times \mathbf{F} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y\rangle = \langle 1 - 1, 0 - 0, 0 - (-2y)\rangle = \langle 0, 0, 2y\rangle = \langle 0, 0, 2t\rangle$ while $\mathbf{n} = (d\mathbf{r}/ds) \times (d\mathbf{r}/dt) = \langle 1, 0, t\rangle \times \langle 0, 1, s\rangle = \langle -t, -s, 1\rangle$ which has correct orientation since z-coordinate is positive. Then $(\nabla \times \mathbf{F}) \cdot \mathbf{n} = 2t$ so by Stokes's theorem the circulation equals the surface integral $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\,d\sigma = \int_0^1 \int_0^2 2t\,dt\,ds = \boxed{4}$.

9. (a) Use the divergence theorem: $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \operatorname{div}(\mathbf{F})\,dV$. The solid is $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$ and $\operatorname{div}(\mathbf{F}) = y^2z^2$. Thus by the divergence theorem, the flux is $\iiint_D \operatorname{div}(\mathbf{F})\,dV = \int_0^1\int_0^1\int_0^1 y^2z^2\,dz\,dy\,dx = \boxed{1/9}$.

(b) Use the divergence theorem: $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \operatorname{div}(\mathbf{F})\,dV$. The solid is $0 \le r \le 2$, $0 \le \theta \le 2\pi$, $1 \le z \le 3$ in cylindrical and also $\operatorname{div}(\mathbf{F}) = 3x^2z + 3y^2z = 3r^2z$. Thus by the divergence theorem, the flux is $\iiint_D \operatorname{div}(\mathbf{F})\,dV = \int_0^{2\pi}\int_0^2\int_1^3 3r^2z\cdot r\,dz\,dr\,d\theta = \boxed{96\pi}$.

(c) Use the divergence theorem: $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \operatorname{div}(\mathbf{F})\,dV$. The solid is $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi$, $0 \le \rho \le 1$ in spherical and also $\operatorname{div}(\mathbf{F}) = y^2 + z^2 + x^2 = \rho^2$. Thus by the divergence theorem, the flux is $\iiint_D \operatorname{div}(\mathbf{F})\,dV = \int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho^2\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \boxed{4\pi/5}$.

(d) Use Stokes's theorem: $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma = \oint_C P\,dx + Q\,dy + R\,dz$ where C is the boundary of the hemisphere. We can parametrize the boundary by $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 0\rangle$ for $0 \le t \le 2\pi$. Then $dx = -3\sin t\,dt$, $dy = 3\cos t\,dt$, $dz = 0$ and $P = 6\sin t$, $Q = 6\cos t$, $R = 3\cos t\cdot e^{3\sin t}$. Thus by Stokes, the integral is $\int_0^{2\pi}(-6\sin t)\cdot(-3\sin t)\,dt + (6\cos t)\cdot 3\cos t\,dt + 3\cos t\cdot e^{3\sin t}\cdot 0\,dt = \int_0^{2\pi} 18\,dt = \boxed{36\pi}$.

(e) Use the divergence theorem: $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \operatorname{div}(\mathbf{F})\,dV$. The solid is $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi/4$, $0 \le \rho \le 1$ in spherical and also $\operatorname{div}(\mathbf{F}) = 12$. Thus by the divergence theorem, the flux is $\iiint_D \operatorname{div}(\mathbf{F})\,dV = \int_0^{2\pi}\int_0^{\pi/4}\int_0^1 12\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \boxed{4\pi(2-\sqrt{2})}$.

(f) Use Stokes's theorem: $\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$ where S is the portion of the plane inside the triangle. We can parametrize S as $\mathbf{r}(s,t) = \langle s, t, 4-s-2t\rangle$ for $0 \le s \le 2$, $0 \le t \le 1$, and then $\nabla\times\mathbf{F} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y\rangle = \langle 1-(-1), 0-0, 0-(-2y)\rangle = \langle 2,0,2y\rangle$ while $\mathbf{n} = (d\mathbf{r}/ds)\times(d\mathbf{r}/dt) = \langle 1,0,-1\rangle\times\langle 0,1,-2\rangle = \langle 1,2,1\rangle$ (correct orientation since z-coordinate is positive). Then $(\nabla\times\mathbf{F})\cdot\mathbf{n} = 2+2y = 2+2t$, and so the surface integral is $\int_0^2\int_0^1 (2+2t)\,dt\,ds = \boxed{6}$.

(g) Use Stokes's theorem: $\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$ where S is the given portion of the surface. Parametrize S as $\mathbf{r}(s,t) = \langle s, t, s^2t\rangle$ for $0 \le s \le 1$, $0 \le t \le s$, and then $\nabla\times\mathbf{F} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y\rangle = \langle 1-0, 0-0, 2x-2x\rangle = \langle 1,0,0\rangle$ while $\mathbf{n} = (d\mathbf{r}/ds)\times(d\mathbf{r}/dt) = \langle 1,0,2st\rangle\times\langle 0,1,s^2\rangle = \langle -2st, -s^2, 1\rangle$ (correct orientation since z-coordinate is positive). Then $(\nabla\times\mathbf{F})\cdot\mathbf{n} = -2st$ so the surface integral is $\int_0^1\int_0^s -2st\,dt\,ds = \boxed{-1/4}$.

(h) Use the divergence theorem. The surface is not closed. Close it by including the bottom disc with $x^2+y^2 \le 1$ and $z = 0$. The solid is $0 \le r \le 1$, $0 \le \theta \le 2\pi$, $0 \le z \le 1-r^2$ in cylindrical and also $\operatorname{div}(\mathbf{F}) = y^2 + x^2 = r^2$. Thus by the divergence theorem, the flux through the solid is $\iiint_D \operatorname{div}(\mathbf{F})\,dV = \int_0^{2\pi}\int_0^1\int_0^{1-r^2} r^2\,r\,dz\,dr\,d\theta = \pi/6$. For the piece being subtracted, we have a parametrization $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle$ for $0 \le \theta \le 2\pi$ and $0 \le r \le 1$. Then $\mathbf{n} = (d\mathbf{r}/dr)\times(d\mathbf{r}/d\theta) = \langle\cos\theta, \sin\theta, 0\rangle\times\langle -r\sin\theta, r\cos\theta, 0\rangle = \langle 0,0,r\rangle$, but this has the wrong orientation since it must point downward. Then $\mathbf{F}\cdot(-\mathbf{n}) = -r\sqrt{x^2+y^2} = r^2$, and so the surface integral is $\int_0^{2\pi}\int_0^1 -r^2\,dr\,d\theta = -2\pi/3$. The flux through the top is then $\pi/6 - (-2\pi/3) = \boxed{5\pi/6}$.

(i) Use the divergence theorem. As above the surface is not closed. Close it by including the bottom disc with $x^2+y^2 \le 5$ and $z = 0$. The solid is $0 \le \varphi \le \pi/2$, $0 \le \theta \le 2\pi$, $0 \le \rho \le \sqrt{5}$ in spherical and also $\operatorname{div}(\mathbf{F}) = (3x^2+2z^2) + (x^2+2z^2) + 4y^2 = 4(x^2+y^2+z^2) = 4\rho^2$. Thus by the divergence theorem, the flux through the solid is $\iiint_D \operatorname{div}(\mathbf{F})\,dV = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{\sqrt{5}} 4\rho^2\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = 40\pi\sqrt{5}$. For the piece being subtracted, we have a parametrization $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle$ for $0 \le \theta \le 2\pi$ and $0 \le r \le \sqrt{5}$. Then $\mathbf{n} = (d\mathbf{r}/dr)\times(d\mathbf{r}/d\theta) = \langle\cos\theta,\sin\theta,0\rangle\times\langle -r\sin\theta, r\cos\theta, 0\rangle = \langle 0,0,r\rangle$, but this has the wrong orientation since it must point downward. Then $\mathbf{F}\cdot(-\mathbf{n}) = 0$, so the flux through the bottom is zero. Therefore, the flux through the top piece is simply $\boxed{40\pi\sqrt{5}}$.
