

1. Compute the following:

- (a) $\int_C x^2 ds$, where C is the line segment from $(0, 1)$ to $(3, 2)$.
 - (b) $\int_C x dx + y dy$, where C is the circle $x^2 + y^2 = 9$.
 - (c) The integral of $\sqrt{x^2 + y^2}$ on the portion of the helix $\mathbf{r}(t) = \langle \cos 3t, \sin 3t, 4t \rangle$ for $0 \leq t \leq \pi$.
 - (d) The average value of y on the upper half of the circle $x^2 + y^2 = 4$.
 - (e) The work done by $\mathbf{F}(x, y, z) = \langle yz, xz, x^3 \rangle \mathbf{N}$ on a particle that travels from $(0, 0, 0)$ to $(1, 1, 1)$ along the curve parametrized by $\mathbf{r}(t) = \langle t^2, t^4, t^3 \rangle \mathbf{m}$.
 - (f) The (outward normal) flux of $\mathbf{F}(x, y) = \langle y + x, y - x \rangle$ across, and the circulation of \mathbf{F} along, the path C along the upper half of $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$.
 - (g) The (outward normal) flux of $\mathbf{F}(x, y) = \langle xy, y^2 \rangle$ across, and the circulation of \mathbf{F} along, the curve $\mathbf{r}(t) = \langle t^2, t^3 \rangle$ for $0 \leq t \leq 1$.
 - (h) $\int_C (x^2 + y^2 + z^3) ds$, where C is the curve parametrized by $\mathbf{r}(t) = \langle \cos \pi t, \sin \pi t, 2t \rangle$ for $0 \leq t \leq 2$.
 - (i) The work done by $\mathbf{F}(x, y, z) = \langle 5, z^2, -3x \rangle$ on a particle with position $\mathbf{r}(t) = \langle \sin t, 3t, \cos t \rangle$ at time t , from $t = 0$ to $t = 2\pi$.
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2. Let C be the upper half of the ellipse $x^2 + 4y^2 = 4$, traversed counterclockwise from $(2, 0)$ to $(-2, 0)$.

- (a) Find a parametrization of C .
 - (b) Find the counterclockwise circulation $\int_C \mathbf{F} \cdot \mathbf{T} ds$ of $\mathbf{F}(x, y) = \langle -y, x + 2y \rangle$ along the curve C .
 - (c) Find the outward normal flux $\int_C \mathbf{F} \cdot \mathbf{N} ds$ of $\mathbf{F}(x, y) = \langle 2y^2, xy \rangle$ across the curve C .
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3. Find an equation for the tangent plane to each surface at the given point:

- (a) The surface parametrized by $\mathbf{r}(s, t) = \langle s + t, s^2 + t^2, s^3 + t^3 \rangle$ where $s = 1$ and $t = 2$.
 - (b) The surface parametrized by $\mathbf{r}(s, t) = \langle s^2, 2st, 3t^3 \rangle$ at the point $(x, y, z) = (1, 2, -3)$.
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4. Parametrize each surface, and then set up (do not evaluate) an iterated double integral for each of the following:

- (a) $\iint_S (x^2 + y^2) d\sigma$ where S is the portion of the plane $z = 2x + 2y$ above the rectangle with $1 \leq x \leq 2$, $2 \leq y \leq 4$.
 - (b) $\iint_S (zx^2 + zy^2) d\sigma$ where S is the portion of the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = 4$.
 - (c) $\iint_S \sqrt{x^2 + y^2} d\sigma$ where S is the portion of the cone $z = 4\sqrt{x^2 + y^2}$ below $z = 8$.
 - (d) $\iint_S z^2 d\sigma$ where S is the unit sphere.
 - (e) $\iint_S x d\sigma$ where S is the portion of the paraboloid $z = x^2 + y^2$ where $0 \leq x \leq 1$ and $0 \leq y \leq 1$.
 - (f) $\iint_S \sqrt{z} d\sigma$ where S is the portion of the paraboloid $z = x^2 + y^2$ where $z \leq 9$.
 - (g) $\iint_S y d\sigma$ where S is the portion of the sphere $x^2 + y^2 + z^2 = 9$ with $x, y, z \geq 0$.
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5. Set up (do not evaluate) an iterated double integral giving the area of each surface:

- (a) The portion of the plane $z = 3x + 4y + 11$ above the region with $0 \leq x \leq 1$ and $0 \leq y \leq 2$.
 - (b) The portion of the surface parametrized by $\mathbf{r}(s, t) = \langle s^2, st, t^2 \rangle$ with $0 \leq s \leq 1$ and $0 \leq t \leq 2$.
 - (c) The portion of the cone $z = 3\sqrt{x^2 + y^2}$ that lies between the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.
 - (d) The portion of the sphere $x^2 + y^2 + z^2 = 9$ that lies below the cone $z = \sqrt{3} \cdot \sqrt{x^2 + y^2}$.
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6. For each vector field \mathbf{F} and surface S , compute the outward normal flux $\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma$ of \mathbf{F} across S :

- (a) $\mathbf{F} = \langle x^2, 2x^2, 3x^2 \rangle$, where S is the portion of the plane $x + y + z = 4$ with $0 \leq x \leq 1$ and $0 \leq y \leq 2$, oriented upwards.
 - (b) $\mathbf{F} = \langle xz, yz, x^2 + y^2 \rangle$, where S is the portion of the cone $z = \sqrt{x^2 + y^2}$ inside the cylinder $x^2 + y^2 = 6$, oriented upwards.
 - (c) $\mathbf{F} = \langle -xz, -yz, z^4 \rangle$, where S is the portion of the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = 3$, oriented outwards.
 - (d) $\mathbf{F} = \langle y, -x, z \rangle$, where S is the portion of the surface $z = x^2 + y^2$ below $z = 2$, oriented upwards.
 - (e) $\mathbf{F} = \langle x, y, z \rangle$, where S is the upper half of the unit sphere, oriented upwards.
 - (f) $\mathbf{F} = \langle y^3, x^3, 1 \rangle$, where S is the portion of the paraboloid $z = 1 - x^2 - y^2$ above the xy -plane, oriented upwards.
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7. For each vector field, (i) find the divergence and curl of \mathbf{F} , and (ii) determine whether \mathbf{F} is conservative and (if so) find a potential function U :

- (a) $\mathbf{F} = \langle x^3 + xy, y^3 + xy, 0 \rangle$.
 - (b) $\mathbf{F} = \langle yz + 2x, xz + 2z, xy + 2y \rangle$.
 - (c) $\mathbf{F} = \langle x^2yz, x^2z^2, 2x^2yz \rangle$.
 - (d) $\mathbf{F} = \langle 2xyz + e^z, x^2z + 3, xe^z + x^2y + 2 \rangle$.
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8. Evaluate $\int_C yz \, dx + xz \, dy + xy \, dz$, where C is the curve $\mathbf{r}(t) = \langle te^t, \arctan(t), \ln(1+t) \rangle$ for $0 \leq t \leq 1$.

9. A particle moves through the force field $\mathbf{F}(x, y) = \langle 9y - e^{x^2} + 11, 9x + \sin \sqrt{y} \rangle$.

- (a) Is the vector field \mathbf{F} conservative? Explain why or why not.
 - (b) Calculate the work done by \mathbf{F} if the particle starts at $(0, 0)$, moves counterclockwise along the circle $x^2 + y^2 = 16$ to $(-4, 0)$, then finally moves in a straight line back to $(0, 0)$.
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