1. Find the minimum and maximum values of f (and all points where they occur) on the given region:
   (a) $f(x,y) = x^2 - 2xy + 3y^2 - 4y$ on the triangle with vertices $(0,0)$, $(2,0)$, and $(2,4)$.
   (b) $f(x,y) = x + y$ on the circular region $x^2 + y^2 \leq 4$.
   (c) $f(x,y) = 2x^2 - y$ on the region below $y = 8x$ and above $y = x^2$.

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2. Find the minimum and maximum values of f (and all points where they occur) subject to the given constraint:
   (a) $f(x,y) = x + 3y + 2$ subject to $x^2 + y^2 = 40$.
   (b) $f(x,y) = xy^2$ subject to $x^2 + y^2 = 12$.
   (c) $f(x,y) = xy$ subject to $3x + y = 60$.
   (d) $f(x,y,z) = 2x + 4y + 5z$ subject to $x^2 + y^2 + z^2 = 1$.
   (e) $f(x,y,z) = xyz$ subject to $x^2 + 4y^2 + 16z^2 = 16$.

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3. You have 60 meters of fencing and wish to make a rectangular enclosure along a straight river, meaning that you only need to fence the east, west, and north sides (not the south side). What dimensions maximize the total area of the enclosure?

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4. Evaluate the following double integrals:
   (a) $\int_0^2 \int_y^{2y} xy^2 \, dx \, dy$.
   (b) $\int_0^1 \int_{x^3}^{x^2} x \, dy \, dx$.
   (c) $\int_{\pi/3}^{\pi/2} \int_1^2 r \sin(\theta) \cdot r \, dr \, d\theta$.

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5. Set up (but do not evaluate) integrals for the following, using both integration orders $dy\,dx$ and $dx\,dy$:
   (a) The integral of $x^2 y$ on the region $0 \leq x \leq 1$, $0 \leq y \leq 3$.
   (b) $\iint_R (x+y)\,dA$ on the region R between the curves $y = 8\sqrt{x}$ and $y = x^2$.
   (c) The volume under $z = x^3$ above the triangle in the xy-plane with vertices $(0,0)$, $(1,1)$, and $(2,0)$.
   (d) The area of the region between the curves $y = x^2 - 1$ and $y = 5$.

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6. Reverse the order of integration for each of the following integrals:
   (a) $\int_0^3 \int_0^{x^2} xy \, dy \, dx$.
   (b) $\int_1^2 \int_y^{y^2} y^4 \, dx \, dy$.

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7. Evaluate the double integral $\int_0^8 \int_{x/2}^4 \frac{e^y}{y} \, dy \, dx$ by reversing the order of integration.

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8. Set up, and then evaluate, the following integrals in polar coordinates:
   (a) The integral of $f(x,y) = x$ on the region inside $x^2 + y^2 = 1$ with $x \leq 0$ and $y \leq 0$.
   (b) $\iint_R \sqrt{x^2 + y^2}\,dA$ where R is the region inside $x^2 + y^2 = 16$, above $y = x$ and $y = -x$.
   (c) The volume under $z = 4 - x^2 - y^2$ and above the xy-plane.

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9. Evaluate the double integral $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{x^2+y^2}}\, dy\, dx$ by converting it to polar coordinates.

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10. Evaluate each of the following triple integrals:

(a) $\displaystyle\int_0^2 \int_x^{2x} \int_x^y 6z\, dz\, dy\, dx.$

(b) $\displaystyle\int_0^{\pi} \int_0^{\pi} \int_0^2 \rho^3 \sin\varphi\, d\rho\, d\varphi\, d\theta.$

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11. Compute the following integrals by converting to cylindrical or spherical coordinates:

(a) $\displaystyle\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_0^{4y} \sqrt{x^2+y^2}\, dz\, dy\, dx.$

(b) $\displaystyle\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx.$

(c) $\displaystyle\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-1}^{x^2+y^2} \frac{1}{\sqrt{x^2+y^2}} dz\, dy\, dx.$

(d) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} \frac{z^2}{\sqrt{x^2+y^2+z^2}}\, dz\, dy\, dx.$

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12. Set up (but do not evaluate) triple integrals for the following:

(a) The integral of xyz on the region above $z = y^2$, below $z = 9$, between $x = 1$ and $x = 2$.
(b) $\iiint_D (x^2+y^2)\, dV$ on the region D above $z = x^2+y^2$, below $z = 7$, for $0 \le x \le 1$ and $0 \le y \le 2$.
(c) The integral of $z\sqrt{x^2+y^2}$ on the region with $x \le 0$, inside $x^2+y^2 = 4$, above $z = 0$, below $y + z = 4$.
(d) $\iiint_D (x^2+y^2+z^2)\, dV$ on the region D above $z = 2\sqrt{x^2+y^2}$ and below $z = 3$.
(e) The volume of the region bounded by $z = 2x$, $z = 3x$, $y = 1$, $y = 2$, $x = y$, and $x = 2y$.
(f) The integral of $\sqrt{x^2+y^2+z^2}$ on the region above $z = -\sqrt{3(x^2+y^2)}$ and inside $x^2+y^2+z^2 = 4$.
(g) The volume of the solid below $z = 5 - x^2 - y^2$, above the xy-plane, and outside $x^2+y^2 = 1$.
(h) The average value of $x^2+y^2+z^2$ on the portion of $x^2+y^2+z^2 \le 4$ inside the first octant (with $x, y, z \ge 0$).
(i) The integral of x on the region with $x \ge 0$, $y \ge 0$, $z \ge 0$ and below $z = 4 - x - y^2$.

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13. Find the total mass and the coordinates of the center of mass for each object:

(a) The solid bounded by $0\text{cm} \le x \le 1\text{cm}$, $0\text{cm} \le y \le 2\text{cm}$, $0\text{cm} \le z \le 3\text{cm}$ with density $d(x,y,z) = z\,\text{g}/\text{cm}^3$.
(b) The solid bounded by $0 \le z \le \sqrt{x^2+y^2} \le 2$ with density $d(x,y,z) = 2\sqrt{x^2+y^2}$.
(c) The solid between $x^2+y^2+z^2 = 2$ and $x^2+y^2+z^2 = 3$ with density $d(x,y,z) = 3(x^2+y^2+z^2)^{3/2}\text{kg}/\text{m}^3$.
(d) The solid between $z = \sqrt{x^2+y^2}$ and $z = \sqrt{3(x^2+y^2)}$ inside $x^2+y^2+z^2 = 9$ with density $d(x,y,z) = 1$.
