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1. (a) Point list is $(1,1)$, $(0,0)$, $(2,0)$, $(2,4/3)$, $(2,4)$, $(4/9,8/9)$. Min of -2 at $(1,1)$, max of 20 at $(2,4)$.
   (b) Point list is $(\sqrt{2},\sqrt{2})$, $(-\sqrt{2},-\sqrt{2})$. Min of $-2\sqrt{2}$ at $(-\sqrt{2},-\sqrt{2})$, max of $2\sqrt{2}$ at $(\sqrt{2},\sqrt{2})$.
   (c) Point list is $(0,0)$, $(8,64)$, $(2,16)$. Min of -8 at $(2,16)$, max of 64 at $(8,64)$.

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2. (a) System is $1=2\lambda x$, $3=2\lambda y$, $x^2+y^2=40$. Thus $x=1/(2\lambda)$, $y=3/(2\lambda)$, so $10/(4\lambda^2)=40$ so $\lambda=\pm 1/4$, yielding $(x,y)=(2,6),(-2,-6)$. Min of -18 at $(-2,-6)$, max of 22 at $(2,6)$.
   (b) System is $y^2=2\lambda x$, $2xy=2\lambda y$, $x^2+y^2=12$. If $y=0$ then get points $(\pm\sqrt{12},0)$. Otherwise, second equation gives $x=\lambda$, and then first equation gives $y^2=2\lambda^2$, so third equation is $3\lambda^2=12$ so $\lambda=\pm 2$ and $(x,y)=(\pm 2,\pm\sqrt{8})$. Min of -16 at $(-2,\pm\sqrt{8})$, max of 16 at $(2,\pm\sqrt{8})$.
   (c) System is $y=3\lambda$, $x=\lambda$, $3x+y=60$. Thus $6\lambda=60$ so $\lambda=10$ yielding $(x,y)=(10,30)$. Minimum does not exist (f goes to $-\infty$ for large x or large y), maximum is 300 at $(10,30)$.
   (d) System is $2=2\lambda x$, $4=2\lambda y$, $5=2\lambda z$, $x^2+y^2+z^2=1$. Thus $x=2/(2\lambda)$, $y=4/(2\lambda)$, $z=5/(2\lambda)$, so $45/(4\lambda^2)=1$ so $\lambda=\pm\sqrt{45/4}$, yielding $(x,y,z)=\pm\frac{1}{\sqrt{45}}(2,4,5)$. Min of $-\sqrt{45}$ at $-\frac{1}{\sqrt{45}}(2,4,5)$, max of $\sqrt{45}$ at $\frac{1}{\sqrt{45}}(2,4,5)$.
   (e) System is $yz=2\lambda x$, $xz=8\lambda y$, $xy=32\lambda z$, $x^2+4y^2+16z^2=48$. If one variable is zero then so must be a second, yields points $(x,y,z)=(\pm\sqrt{48},0,0)$, $(0,\pm\sqrt{12},0)$, $(0,0,\pm\sqrt{3})$. If none are zero, divide first two equations to get $y/x=x/(4y)$ so $x/y=\pm 2$. Similarly, $x/z=\pm 4$. Yields points $(\pm 4,\pm 2,\pm 1)$ with all sign choices. Min of -8 and max of 8.

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3. If the length l is east-west and the width w is north-south, then we maximize $A=lw$ subject to $2l+w=60$m. Using Lagrange gives $w=2\lambda$, $l=\lambda$, $2l+w=60$m and so $4\lambda=60$ and $\lambda=15$. Thus $l=15$m, $w=30$m.

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4. (a) $I=\int_0^2 \frac{3}{2}y^4\,dy=\frac{48}{5}$. (b) $I=\int_0^1(x^3-x^4)\,dx=\frac{1}{20}$. (c) $I=\int_{\pi/3}^{\pi/2}\frac{7}{3}\sin\theta\,dx=\frac{7}{6}$.

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5. (a) Integrals are $\int_0^1\int_0^3 x^2y\,dy\,dx$ and $\int_0^3\int_0^1 x^2y\,dx\,dy$.
   (b) Integrals are $\int_0^4\int_{x^2}^{8\sqrt{x}}(x+y)\,dy\,dx$ and $\int_0^{16}\int_{y^2/64}^{\sqrt{y}}(x+y)\,dx\,dy$.
   (c) Integrals are $\int_0^1\int_0^x x^3\,dy\,dx+\int_1^2\int_0^{2-x}x^3\,dy\,dx$ and $\int_0^1\int_y^{2-y}x^3\,dx\,dy$.
   (d) Integrals are $\int_{-\sqrt{6}}^{\sqrt{6}}\int_{x^2-1}^5 1\,dy\,dx$ and $\int_{-1}^5\int_{-\sqrt{y+1}}^{\sqrt{y+1}}1\,dx\,dy$.

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6. (a) $\int_0^9\int_{\sqrt{y}}^3 xy\,dx\,dy$. (b) $\int_1^2\int_{\sqrt{x}}^x y^4\,dy\,dx+\int_2^4\int_{\sqrt{x}}^2 y^4\,dy\,dx$.

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7. Region is interior of a triangle. Reversed integral is $\int_0^4\int_0^{2y}\frac{e^y}{y}\,dx\,dy=\int_0^4 2e^y\,dy=2(e^4-1)$.

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8. (a) Region is interior of a quarter-circle. Integral is $\int_\pi^{3\pi/2}\int_0^1(r\cos\theta)\,r\,dr\,d\theta=\int_\pi^{3\pi/2}\frac{1}{3}\cos\theta\,d\theta=-\frac{1}{3}$.
   (b) Region is interior of a quarter-circle. Integral is $\int_{\pi/4}^{3\pi/4}\int_0^4(r)\,r\,dr\,d\theta=\int_{\pi/4}^{3\pi/4}\frac{64}{3}\,d\theta=\frac{32\pi}{3}$.
   (c) Region is interior of a circle. Integral is $\int_0^{2\pi}\int_0^2(4-r^2)\,r\,dr\,d\theta=\int_0^{2\pi}12\,d\theta=24\pi$.

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9. Region is interior of a quarter-circle. Polar integral is $\int_0^{\pi/2}\int_0^1 \frac{1}{r}\cdot r\,dr\,d\theta = \int_0^{\pi/2}\int_0^1 1\,dr\,d\theta = \int_0^{\pi/2} 1\,d\theta = \pi/2$.

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10. (a) $I = \int_0^2\int_x^{2x} 3(y^2-x^2)\,dy\,dx = \int_0^2 4x^3\,dx = 16$. (b) $I = \int_0^{\pi}\int_0^{\pi} 4\sin\varphi\,d\varphi\,d\theta = \int_0^{\pi} 8\,d\theta = 8\pi$.

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11. (a) Cylindrical: $\int_0^{\pi}\int_0^1\int_0^{4r\sin\theta} r\cdot r dz\,dr\,d\theta = \int_0^{\pi}\int_0^1 r^3\cdot 4\sin\theta\,dr\,d\theta = \int_0^{\pi}\sin\theta\,d\theta = 2$.
    (b) Spherical: $\int_0^{\pi}\int_0^{\pi/2}\int_0^2 \rho\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \int_0^{\pi}\int_0^{\pi/2} 4\sin\varphi\,d\rho\,d\varphi\,d\theta = \int_0^{\pi/2} 4\,d\theta = 2\pi$.
    (c) Cylindrical: $\int_{-\pi/2}^{\pi/2}\int_0^3\int_{-1}^{r^2}\frac{1}{r}\cdot r\,dz\,dr\,d\theta = \int_{-\pi/2}^{\pi/2}\int_0^3 (r^2+1)\,dr\,d\theta = \int_{-\pi/2}^{\pi/2} 12\,d\theta = 12\pi$.
    (d) Spherical: $\int_0^{2\pi}\int_0^{\pi/4}\int_0^2 \frac{\rho^2\cos^2\varphi}{\rho}\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \int_0^{2\pi}\int_0^{\pi/4} 4\cos^2\varphi\sin\varphi\,d\rho\,d\varphi\,d\theta = 4\pi(4-\sqrt{2})/3$.

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12. (a) Rectangular: $\int_1^2\int_{-3}^3\int_{y^2}^9 xyz\,dz\,dy\,dx$.
    (b) Rectangular: $\int_0^1\int_0^2\int_{x^2+y^2}^7 (x^2+y^2)\,dz\,dy\,dx$.
    (c) Cylindrical: $\int_{\pi/2}^{3\pi/2}\int_0^2\int_0^{4-r\sin\theta} zr\cdot r\,dz\,dr\,d\theta$.
    (d) Cylindrical: $\int_0^{2\pi}\int_0^{3/2}\int_{2r}^3 (r^2+z^2)\,r\,dz\,dr\,d\theta$.
    (e) Rectangular: $\int_1^2\int_y^{2y}\int_{2x}^{3x} 1\,dz\,dx\,dy$.
    (f) Spherical: $\int_0^{2\pi}\int_0^{5\pi/6}\int_0^2 \rho\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$.
    (g) Cylindrical: $\int_0^{2\pi}\int_1^{\sqrt{5}}\int_0^{5-r^2} 1\cdot r\,dz\,dr\,d\theta$.
    (h) Spherical: $\frac{1}{4\pi/3}\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 \rho^2\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$.
    (i) Rectangular: $\int_0^4\int_0^{\sqrt{4-x}}\int_0^{4-x-y^2} x\,dz\,dy\,dx$.

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13. (a) Mass is $M = \int_0^1\int_0^2\int_0^3 z\,dz\,dy\,dx = 9$g. Moments are $M_x = \int_0^1\int_0^2\int_0^3 xz\,dz\,dy\,dx = \frac{9}{2}$g, $M_y = \int_0^1\int_0^2\int_0^3 yz\,dz\,dy\,dx =$ 9g, $M_z = \int_0^1\int_0^2\int_0^3 z\cdot z\,dz\,dy\,dx = 18$g. Center of mass is $\frac{1}{M}(M_x, M_y, M_z) = (1/2, 1, 2)$cm.
    (b) Mass is $M = \int_0^{2\pi}\int_0^2\int_0^r 2r\cdot r\,dz\,dr\,d\theta = 128\pi/5$. Moments are $M_x = \int_0^{2\pi}\int_0^2\int_0^r r\cos\theta\cdot 2r\cdot r\,dz\,dr\,d\theta = 0$, $M_y = \int_0^{2\pi}\int_0^2\int_0^r r\sin\theta\cdot 2r\cdot r\,dz\,dr\,d\theta = 0$, $M_z = \int_0^{2\pi}\int_0^2\int_0^r z\cdot 2r\cdot r\,dz\,dr\,d\theta = 64\pi/3$. Center of mass is $\frac{1}{M}(M_x, M_y, M_z) = (0, 0, 5/6)$.
    (c) Mass is $M = \int_0^{2\pi}\int_0^{\pi}\int_{\sqrt{2}}^{\sqrt{3}} 3\rho^3\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = 38\pi$kg. Center of mass is $(0,0,0)$m by symmetry. Alternatively, could evaluate moments as integrals: $M_x = \int_0^{2\pi}\int_0^{\pi}\int_{\sqrt{2}}^{\sqrt{3}} \rho\cos\theta\sin\varphi\cdot 3\rho^3\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = 0$, $M_y = \int_0^{2\pi}\int_0^{\pi}\int_{\sqrt{2}}^{\sqrt{3}} \rho\sin\theta\sin\varphi\cdot 3\rho^3\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = 0$, $M_z = \int_0^{2\pi}\int_0^{\pi}\int_{\sqrt{2}}^{\sqrt{3}} \rho\cos\varphi\cdot 3\rho^3\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = 0$.
    (d) Mass is $M = \int_0^{2\pi}\int_{\pi/6}^{\pi/4}\int_0^3 1\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = 9(\sqrt{3}-\sqrt{2})\pi$. The x and y-coordinates of the center of mass are 0 by symmetry. The z-moment is $M_z = \int_0^{2\pi}\int_{\pi/6}^{\pi/4}\int_0^3 \rho\cos\varphi\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = 81\pi/16$. Center of mass is $\frac{1}{M}(M_x, M_y, M_z) = (0, 0, \frac{9\sqrt{3}+9\sqrt{2}}{16})$.
