1. (a) $\hat{\mu}_X = 8.8$, $S_X = 2.77489$.
   (b) $n-1 = 4$ df. 80% CI: $t_{\alpha/2} = 1.5332$ yielding $(6.89734, 10.70266)$. 95% CI: $t_{\alpha/2} = 2.7764$ yielding $(5.35352, 12.24548)$.
   (c) One-sample t test. $H_0 : \mu = 4$, $H_a : \mu > 4$. Test statistic $t = 3.86795$, p-value is $P(T_4 \geq 3.86795) = 0.00901$, reject null hypothesis.
   (d) $n-1 = 4$ df. 80% CI: $\chi^2_{\alpha/2} = 1.0636$, $\chi^2_{1-\alpha/2} = 7.7794$ yielding $(3.95915, 28.95762)$. 95% CI: $\chi^2_{\alpha/2} = 0.4844$, $\chi^2_{1-\alpha/2} = 11.1433$ yielding $(2.76400, 63.58138)$.
   (e) χ^2 test for variance. $H_0 : \sigma^2 = 100$, $H_a : \sigma^2 < 100$. Test statistic $\chi^2 = 0.308$, p-value is $P(Q_4 \leq 0.308) = 0.01070$, reject null hypothesis.
   (f) Student's equal-variances t test. $H_0 : \mu_X - \mu_Y = 0$, $H_a : \mu_X - \mu_Y \neq 0$. $\hat{\mu}_X = 8.8$, $\hat{\mu}_Y = 7$, $S_X = 2.77489$, $S_Y = 5.59762$, $S_{\text{pool}} = 4.22239$. Test statistic $t = 0.63549$ with $df = 7$, p-value is $p = 2P(T_7 \geq 0.63549) = 0.54532$, fail to reject null hypothesis.
   (g) 7 df, use pooled variance. $\hat{\mu}_X - \hat{\mu}_Y = 1.8$, $S = S_{\text{pool}}\sqrt{\frac{1}{5} + \frac{1}{4}} = 2.83246$. 50% CI: $t_{\alpha/2} = 0.7111$ yielding $(-0.21428, 3.81428)$. 90% CI: $t_{\alpha/2} = 1.89458$ yielding $(-3.56633, 7.16633)$.

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2. (a) One-sample t test (z test is also acceptable since sample size is large). $H_0 : \mu = 20.000$, $H_a : \mu \neq 20.000$. Test statistic $t = -0.62827$ with $n - 1 = 99$ df, so p-value is $p = 2P(T_{99} \leq -0.62827) = 0.53127$ [for z-test, p-value is $2P(N_{0,1} \leq -0.62827) = 0.52983$], fail to reject null hypothesis. (Bolt diameter at spec.)
   (b) $n-1 = 99$ df. 90% CI: $t_{\alpha/2} = 1.6604$ yielding (19.9563mm, 20.0197mm). 99% CI: $t_{\alpha/2} = 2.6364$ yielding (19.9378mm, 20.0382mm). [Using z instead of t gives CIs (19.9566mm, 20.0194mm), (19.9388mm, 20.0372mm).
   (c) χ^2 test for variance. $H_0 : \sigma^2 = 0.0400\text{mm}^2$, $H_a : \sigma^2 < 0.0400\text{mm}^2$. $n - 1 = 99$ df. Test statistic $\chi^2 = (n-1)S^2/0.0400 = 90.2905$, p-value is $P(Q_{99} \leq 90.2905) = 0.27748$, fail to reject null hypothesis. (Bolt variance possibly not within spec.)
   (d) $n-1 = 99$ df. 90% CI: $\chi^2_{\alpha/2} = 77.0463$, $\chi^2_{1-\alpha/2} = 123.2252$ yielding (0.1712mm, 0.2165mm). 99% CI: $\chi^2_{\alpha/2} = 66.5101$, $\chi^2_{1-\alpha/2} = 138.9868$ yielding (0.1612mm, 0.2330mm).

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3. These are all t confidence intervals. Use the t-table with $n-1$ degrees of freedom to get $t_{\alpha/2,n}$, measuring the number of standard deviations in the margin of error.
   (a) $\mu = 200.74$, $S = 22.6166$, 50%: $(192.09, 209.39)$, 80%: $(182.22, 219.26)$, 90%: $(174.13, 227.35)$, 99%: $(134.69, 266.79)$.
   (b) $\mu = 74.2$, $S = 3.4254$, 50%: $(73.44, 74.96)$, 80%: $(72.70, 75.70)$, 90%: $(72.21, 76.19)$, 99%: $(70.68, 77.72)$.
   (c) $\mu = 134.33$, $S = 14.0119$, 50%: $(127.73, 140.94)$, 80%: $(119.08, 149.59)$, 90%: $(110.71, 157.96)$, 99%: $(54.04, 214.62)$.
   (d) $\mu = 109333$, $S = 23459$, 50%: $(98000, 120000)$, 80%: $(84000, 135000)$, 90%: $(70000, 149000)$, 99%: $(-25000, 244000)$.
   (e) $\mu = 201000$, $S = 55323$, 50%: $(189000, 231000)$, 80%: $(165000, 255000)$, 90%: $(145000, 275000)$, 99%: $(48000, 372000)$.
   (f) $\mu = 47000$, $S = 19937$, 50%: $(40000, 54000)$, 80%: $(33000, 61000)$, 90%: $(28000, 66000)$, 99%: $(6000, 88000)$.

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4. (a) Two-sample t test. $\hat{\mu}_E = 32$pp, $S_E = 13.6626$pp, $\hat{\mu}_D = 25$pp, $S_D = 6.2849$pp. $H_0 : \mu_E - \mu_D = 0$, $H_a : \mu_E - \mu_D \neq 0$.
   Student: $S_{\text{pool}} = 10.12776$, $df = 7$, test statistic is $t = 1.03034$, p-value is $2P(T_7 \geq 1.03034) = 0.33713$, fail to reject H_0.
   Welch: $S_{\text{unpool}} = 7.29155$, $df = 4.0154$, test statistic is $t = 0.94762$, p-value is $2P(T_{4.0154} \geq 0.94762) = 0.39679$, fail to reject H_0.
   (b) Two-sample t test. $\hat{\mu}_m = 72.2$, $S_m = 14.48012$, $\hat{\mu}_p = 85.16667$, $S_p = 6.70572$. $H_0 : \mu_m - \mu_p = 0$, $H_a : \mu_m - \mu_p < 0$.
   Student: $S_{\text{pool}} = 10.87113$, $df = 9$, test statistic is $t = -1.96978$, p-value is $P(T_9 \leq -1.96978) = 0.04019$, fail to reject H_0.
   Welch: $S_{\text{unpool}} = 7.03096$, $df = 5.4189$, test statistic is $t = -1.84422$, p-value is $P(T_{5.4189} \leq -1.84422) = 0.05998$, fail to reject H_0.

(c) Two-sample t test. $\hat{\mu}_m = 134.\overline{3}$, $S_m = 14.0119$, $\hat{\mu}_b = 181.\overline{6}$, $S_b = 9.6090$. $H_0 : \mu_m - \mu_b = 0$, $H_a : \mu_m - \mu_b < 0$.
Student: $S_{\text{pool}} = 12.01388$, $df = 4$, test statistic is $t = -4.82536$, p-value is $P(T_4 \leq -4.82536) = 0.004245$, reject H_0.
Welch: $S_{\text{unpool}} = 9.80929$, $df = 3.5405$, test statistic is $t = -4.82536$, p-value is $P(T_{3.5405} \leq -4.82536) = 0.005734$, reject H_0.

(d) One-sample t test (matched pairs). $H_0 : \mu_d = 0$, $H_a : \mu_d \neq 0$. $\hat{\mu}_d = 1$, $S_d = 4.95696$, test statistic is $t = 0.57060$, p-value is $P(T_7 \geq 0.57060) = 0.58611$, fail to reject H_0.

(e) χ^2 test for independence. Comparison table is below. χ^2 statistic is $d = 0.54492$, $df = 3$, p-value is $P(Q_3 \geq 0.54492) = 0.90892$, fail to reject H_0 (seems independent).

Exp (Obs)	USA	UK	China	Russia
Senior	45.47 (46)	25.18 (27)	26.31 (26)	21.04 (19)
Junior	75.53 (75)	41.82 (40)	43.69 (44)	34.96 (37)

(f) Two-sample t test. $H_0 : \mu_m - \mu_f = 0$, $H_a : \mu_m - \mu_f > 0$.
Student: $S_{\text{pool}} = \$22139$, $df = 46$, test statistic is $t = 2.58736$, p-value is $p(T_{46} \geq 2.58736) = 0.00644$, reject H_0.
Welch: $S_{\text{unpool}} = \$6770$, $df = 33.137$, test statistic is $t = 2.52268$, p-value is $p(T_{33.137} \geq 2.52268) = 0.00831$, reject H_0.
Pooled CIs: $\hat{\mu}_{\text{diff}} = \$17078$, $S = S_{\text{pool}}\sqrt{\frac{1}{30} + \frac{1}{18}} = \$6601$, $df = 46$. 80% CI: $t_{\alpha/2} = 1.3002$ yielding $(\$8496, \$25660)$, 95% CI: $t_{\alpha/2} = 2.0129$ yielding $(\$3792, \$30364)$.
Unpooled CIs: $\hat{\mu}_{\text{diff}} = \$17078$, $S = S_{\text{unpool}} = \$6770$, $df = 33.137$. 80% CI: $t_{\alpha/2} = 1.3076$ yielding $(\$8226, \$25930)$, 95% CI: $t_{\alpha/2} = 2.0342$ yielding $(\$3307, \$30849)$.

(g) χ^2 test for independence. Comparison table is below. χ^2 statistic is $d = 16.4934$, $df = 1$, p-value is $P(Q_1 \geq 16.4934) = 0.000049$, reject H_0 (seems non-independent).

Exp (Obs)	Callback	No Callback
White	195.5 (234)	2229.5 (2191)
Black	195.5 (157)	2229.5 (2268)

(h) χ^2 test for goodness of fit. Comparison table is below. χ^2 statistic is $d = 112.52$, $df = 2$, p-value is $P(Q_2 \geq 16.4934) = 3.68 \cdot 10^{-25}$, reject H_0 (definitely not as predicted by model!).

Exp (Obs)	0.001	0.01	0.05
Just Over	104 (94)	98 (76)	183 (87)
Just Under	104 (114)	98 (120)	183 (279)

(i) Two-sample t test. $\hat{\mu}_s = \$200.74$, $S_s = \$22.617$, $\hat{\mu}_c = \$196.69$, $S_c = \$20.353$. $H_0 : \mu_s - \mu_c = 0$, $H_a : \mu_s - \mu_c > 0$.
Student: $S_{\text{pool}} = 21.7394$, $df = 5$, test statistic is $t = 0.24392$, p-value is $P(T_5 \geq 0.24392) = 0.40849$, fail to reject H_0.
Welch: $S_{\text{unpool}} = 16.3082$, $df = 4.7206$, test statistic is $t = 0.24834$, p-value is $P(T_{4.7206} \geq 0.24834) = 0.40715$, fail to reject H_0.
Pooled CIs: $\hat{\mu}_{\text{diff}} = \$4.05$, $S = S_{\text{pool}}\sqrt{\frac{1}{30} + \frac{1}{18}} = \$16.604$, $df = 5$. 50% CI: $t_{\alpha/2} = 0.7267$ yielding $(-\$8.02, \$16.12)$, 80% CI: $t_{\alpha/2} = 1.4759$ yielding $(-\$20.46, \$28.56)$.
Unpooled CIs: $\hat{\mu}_{\text{diff}} = \$4.05$, $S = S_{\text{unpool}} = \$16.308$, $df = 4.7206$. 50% CI: $t_{\alpha/2} = 0.7300$ yielding $(-\$7.85, \$15.95)$, 80% CI: $t_{\alpha/2} = 1.4891$ yielding $(-\$20.23, \$28.33)$.

(j) For mean, one-sample t test. $\hat{\mu} = 5.5\%$, $S = 1.5556\%$. $H_0 : \mu = 4\%$, $H_a : \mu > 4\%$, test statistic is $t = 1.92847$, p-value is $P(T_3 \geq 1.92847) = 0.07469$, fail to reject H_0. 95% CI for μ: $t_{\alpha/2} = 3.1824$ yielding $(3.025\%, 7.975\%)$.
For standard deviation, χ^2 test for variance. $H_0 : \sigma = 2.5\%$, $H_a : \sigma < 2.5\%$, test statistic is $\chi^2 = (n-1)S^2/(2.5\%)^2 = 1.1616$, p-value is $P(Q_3 \leq 1.1616) = 0.23777$, fail to reject H_0. 95% CI for σ: $\chi^2_{\alpha/2} = 0.21579$, $\chi^2_{1-\alpha/2} = 9.34840$ yielding $(0.881\%, 5.800\%)$.

(k) χ^2 test for goodness of fit. Expected values are all 100. For (i), χ^2 statistic is $d = 0.42$, $df = 9$, p-value is $P(Q_9 \geq 0.42) = 0.9999857$, fail to reject H_0. For (ii), these do seem to close to the model. Testing for "too good" results gives p-value $P(Q_9 \leq 0.42) = 0.000014$, reject H_0 (model is too accurate, suggests forgery).

(l) χ^2 test for goodness of fit. Expected values are 1000 times Benford probability (i.e., just read as 301, 176, 125, etc.). For (i), χ^2 statistic is $d = 13.4150$, $df = 8$, p-value is $P(Q_8 \geq 13.4150) = 0.09835$, fail to reject H_0. For (ii), these results are not suspiciously accurate – it seems finally that the ex-pollster ex-accountant is doing their job properly!