1. Of 130 students in Math 3081, 88 study every day, 73 work on WeBWorK every day, and 54 do both.
   (a) How many students study every day but don't work on their homework every day?
   (b) How many students neither study nor work on their homework every day?

2. Find the number of 5-letter strings that can be made from the letters ABCDEFG such that:
   (a) The string starts with AG.
   (b) The string has no repeated letters.
   (c) The string contains neither C nor G.
   (d) The string has at least one repeated letter.
   (e) The string contains at least one B.
   (f) The string has no doubled letters (no AA, ...).

3. A tennis team of 14 people selects 3 nonoverlapping pairs of players to make doubles teams. Find the number of ways of making these selections if the order of the 3 pairs (a) matters, (b) does not matter.

4. A fair coin is flipped 10 times. Find the probabilities of the following events:
   (a) All the flips are heads.
   (b) The first and last flips are heads.
   (c) Exactly 4 flips are tails.
   (d) At least 8 heads are obtained.
   (e) The first three flips are all the same.
   (f) There are more tails than heads.

5. Three standard 6-sided dice of different colors are rolled. Find the probabilities of the following events:
   (a) Three of a kind (all dice equal).
   (b) Pair (two dice equal, third different).
   (c) No pair (all dice different).
   (d) 6-3-2 in some order.
   (e) No 6s or 5s are rolled.
   (f) At least one 6 is rolled.
   (g) 3-3-3, given no 6s or 5s are rolled.
   (h) 1-1-5 in some order, given a pair is rolled.
   (i) 6-3-2 in some order, given a 6 is rolled.
   (j) 6-3-2 in some order, given no pair is rolled.

6. An urn contains 10 red and 8 orange balls. 4 balls are drawn without replacement. Find the probabilities that:
   (a) All 4 balls are red.
   (b) 1 ball is red and 3 are orange.
   (c) All 4 balls are red, given that ≥ 1 is red.
   (d) Ball #1 is orange, given that ≥ 3 are red.

7. Suppose A and B are events such that $P(A) = 0.4$, $P(B|A) = 0.8$, and $P(B|A^c) = 0.1$. Find:
   (a) $P(A^c)$.
   (b) $P(A \cap B)$.
   (c) $P(A^c \cap B)$.
   (d) $P(B)$.
   (e) $P(B^c)$.
   (f) $P(A \cup B)$.
   (g) $P(A \cap B^c)$.
   (h) $P(A|B)$.
   (i) $P(B^c|A)$.
   (j) $P(A^c \cap B^c)$.
   (k) $P(A^c|B^c)$.
   (l) $P(A \cup B^c)$.

8. Suppose $P(A) = 0.3$ and $P(B) = 0.4$. Find $P(A \cup B)$ if A and B are (a) mutually exclusive, (b) independent.

9. Market research shows that 15% of Americans and 90% of Canadians like poutine. A math conference has 40% Canadian attendees and 60% American attendees. Find the probabilities that:
   (a) A random attendee is American and likes poutine.
   (b) A random attendee likes poutine.
   (c) A random attendee who likes poutine is American.
   (d) A random poutine-disliking attendee is Canadian.

10. Given that discrete random variables X and Y have probability distributions as below, find:

n	0	1	2	3	4
$P(X = n)$	0.1	0	0.2	0.2	0.5
$P(Y = n)$	0.4	0.1	0.2	0.1	0.2

   (a) $P(1 \leq X \leq 3)$.
   (b) $P(Y > 2)$.
   (c) $E(X)$ and $E(Y)$.
   (d) $E(X + 2Y)$.
   (e) $\text{var}(X)$ and $\sigma(X)$.
   (f) $\text{var}(Y)$ and $\sigma(Y)$.

11. An urn contains 10 pink and 10 green balls. 3 balls are drawn without replacement. If X is the discrete random variable counting the number of green balls selected, find

(a) The probability distribution for X.

(b) $P(X < 3)$.

(c) The expected value of X.

(d) The variance and standard deviation of X.

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12. The continuous random variable X has $E(X) = 11$ and $\sigma(X) = 2$, and the table below contains some selected values of its cdf. Find the following:

z	8	9	10	11	12	13	14	15	16
$P(X \le z)$	0	0.23	0.31	0.45	0.58	0.73	0.79	0.98	1

(a) $P(X < 14)$. (b) $P(9 \le X \le 12)$ (c) $P(X > 13)$. (d) $E(2X + 5)$. (e) $\sigma(2X + 5)$.

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13. Given that discrete random variables X and Y have the joint distribution table given below, find:

$X \setminus Y$	3	4	5
1	0.1	0.1	0
3	0.2	0.2	0.1
4	0	0.1	0.2

(a) $P(X = 1, Y = 4)$.

(b) $P(Y > X)$.

(c) $P(X = Y)$.

(d) $P(X + Y = 7)$.

(e) The marginal distribution of X

(f) The marginal distribution of Y

(g) $P(X > 1)$.

(h) $P(Y < 5)$.

(i) $E(X)$ and $E(Y)$.

(j) $E(X + 2Y)$.

(k) $\text{var}(X)$ and $\sigma(X)$.

(l) $\text{var}(Y)$ and $\sigma(Y)$.

(m) If X, Y are independent.

(n) $\text{cov}(X, Y)$ and $\text{corr}(X, Y)$.

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14. The continuous random variable Z has pdf $p(x) = (8x - x^2)/72$ for $0 \le x \le 6$ and 0 for other x. Find:

(a) $P(1 \le Z \le 4)$.

(b) $P(Z < 1)$ and $P(Z = 1)$.

(c) $P(Z < 3)$ and $P(Z > 3)$.

(d) The c.d.f. (cumulative) for Z.

(e) $E(Z)$ and $E(4Z + 5)$.

(f) $\text{var}(Z)$ and $\sigma(Z)$.

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15. If the continuous random variables X and Y have joint pdf $p(x, y) = c \cdot (x + 3y)$ for $0 \le x \le 3$, $0 \le y \le 2$, find

(a) The value of c.

(b) $P(0 \le X \le 1, 0 \le Y \le 1)$.

(c) $P(X < 2)$.

(d) $P(X < Y)$.

(e) $P(X + Y < 2)$.

(f) $P(X = 1)$.

(g) The marginal distribution of X

(h) The marginal distribution of Y

(i) $E(X)$ and $E(Y)$.

(j) $E(X^2)$ and $E(Y^2)$.

(k) $E(XY)$.

(l) $\text{var}(X)$ and $\sigma(X)$.

(m) $\text{var}(Y)$ and $\sigma(Y)$.

(n) If X, Y are independent.

(o) $\text{cov}(X, Y)$ and $\text{corr}(X, Y)$.

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16. Suppose $E(X) = 1$, $E(Y) = 3$, $E(X^2) = 10$, $E(Y^2) = 13$, and $E(XY) = 4$. Find:

(a) $E(2X - 3)$.

(b) $E(X + 2Y)$.

(c) $E(XY + 2X^2)$.

(d) $\text{var}(X)$ and $\sigma(X)$.

(e) $\text{var}(Y)$ and $\sigma(Y)$.

(f) $\text{cov}(X, Y)$.

(g) $\text{corr}(X, Y)$.

(h) $\text{var}(X + Y)$.

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17. A coin with probability 2/3 of landing heads is flipped 1800 times. Let Y be the random variable counting the total number of heads.

(a) Find exact expressions for $P(Y = 1200)$ and $P(1200 \le Y \le 1250)$ (you need not evaluate them).

(b) Find $E(Y)$ and $\sigma(Y)$.

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18. A basketball player has a 0.8 probability of making a 1-point free throw, a 0.4 probability of making a 2-point shot, and a 0.2 probability of making a 3-point shot. All shots are independently likely to score.

(a) If the player takes ten 2-point shots, what is the probability she scores on at least 2 shots?

(b) Find the expected number, and standard deviation, of total points from 100 free throws.

(c) Find the expected number, and standard deviation, of total points from a 3-point shot plus a 2-point shot.
