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1. (a) $88 - 54 = 34$. (b) $130 - (34 + 73) = 23$.

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2. (a) There are 7 choices for letters 3,4,5 so $1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 = 343$ strings.
(b) There are 7 choices for first letter, 6 for second, etc, so $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$ strings.
(c) There are 5 choices for each letter, so there are $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3125$ strings.
(d) There are 7^5 total strings and this is the complement of (b), so $7^5 - 2520 = 14287$ strings.
(e) There are $6^5 = 7776$ strings without a B, so there are $7^5 - 6^5 = 9031$ strings.
(f) There are 7 choices for first letter and 6 for each other letter, so $7 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 9072$ strings.

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3. (a) There are $\binom{14}{2}$ ways to pick the first pair, $\binom{12}{2}$ ways to pick the second pair, and $\binom{10}{2}$ ways to pick the third pair for a total of $\binom{14}{2} \cdot \binom{12}{2} \cdot \binom{10}{2} = 270270$ possible choices of the three pairs.
(b) The only difference is that the order of the three pairs is now irrelevant. Since they may be permuted in 3! possible, ways, the answer is lowered by a factor of $\frac{1}{3!}$, so there are $\frac{1}{3!} \cdot \binom{14}{2} \cdot \binom{12}{2} \cdot \binom{10}{2} = 45045$ possible choices.

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4. (a) $1/2^{10} = 1/1024$.
(b) $1/4$.
(c) $\binom{10}{4}/2^{10} = 105/512$.
(d) $\frac{1}{2^{10}}[\binom{10}{8} + \binom{10}{9} + \binom{10}{10}] = 7/128$.
(e) $1/4$.
(f) $\frac{1}{2^{10}}[\binom{10}{6} + \binom{10}{7} + \cdots + \binom{10}{10}] = 193/512$.

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5. (a) $6/216 = 1/36$.
(b) $6 \cdot 5 \cdot 3/216 = 5/12$.
(c) $6 \cdot 5 \cdot 4/216 = 5/9$.
(d) $3!/216 = 1/36$.
(e) $4 \cdot 4 \cdot 4/216 = 8/27$.
(f) $1 - \frac{5 \cdot 5 \cdot 5}{216} = 91/216$.
(g) $\frac{1}{216}/\frac{8}{27} = 1/64 = 1/4^3$.
(h) $\frac{3}{216}/\frac{5}{12} = 1/30 = 1/(6 \cdot 5)$.
(i) $\frac{1}{36}/\frac{91}{216} = 6/91$.
(j) $\frac{1}{36}/\frac{5}{9} = 1/20 = 1/\binom{6}{3}$.

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6. (a) $\frac{10}{18} \cdot \frac{9}{17} \cdot \frac{8}{16} \cdot \frac{7}{15} = \frac{7}{102}$.
(b) $4 \cdot \frac{10}{18} \cdot \frac{8}{17} \cdot \frac{7}{16} \cdot \frac{6}{15} = \frac{28}{153}$.
(c) $P(\geq 1 \text{ red}) = 1 - P(\text{none red}) = 1 - \frac{8}{18} \cdot \frac{7}{17} \cdot \frac{6}{16} \cdot \frac{5}{15} = \frac{299}{306}$, and the intersection of these events is drawing 4 reds, so the desired conditional probability is $\frac{7}{102}/\frac{299}{306} = \frac{21}{299}$.
(d) $P(\geq 3 \text{ red}) = \frac{10}{18} \cdot \frac{9}{17} \cdot \frac{8}{16} \cdot \frac{7}{15} + 4 \cdot \frac{8}{18} \cdot \frac{10}{17} \cdot \frac{9}{16} \cdot \frac{8}{15} = \frac{13}{34}$, and the intersection of these events is drawing orange-red-red-red of probability $\frac{8}{18} \cdot \frac{10}{17} \cdot \frac{9}{16} \cdot \frac{8}{15} = \frac{4}{51}$, so the desired conditional probability is $\frac{4}{51}/\frac{13}{34} = \frac{8}{39}$.

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7. (a) $1 - P(A) = 0.6$.
(b) $P(B|A) \cdot P(A) = 0.32$.
(c) $P(B|A^c) \cdot P(A^c) = 0.06$.
(d) $P(A \cap B) + P(A \cap B^c) = 0.38$.
(e) $1 - P(B) = 0.62$.
(f) $P(A) + P(B) - P(A \cap B) = 0.46$.
(g) $P(A) - P(A \cap B) = 0.08$.
(h) $P(A \cap B)/P(B) = 16/19$.
(i) $P(A \cap B^c)/P(A) = 1/5$.
(j) $1 - P(A \cup B) = 0.54$.
(k) $P(A^c \cap B^c)/P(B^c) = 27/31$.
(l) $1 - P(A^c \cap B) = 0.94$.

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8. (a) Mutually disjoint means $P(A \cap B) = 0$, so then $P(A \cup B) = 0.3 + 0.4 - 0 = 0.7$.
(b) Independent means $P(A \cap B) = P(A)P(B) = 0.12$, so then $P(A \cup B) = 0.3 + 0.4 - 0.12 = 0.58$.

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9. (a) $0.15 \cdot 0.6 = 0.09$. (b) $0.15 \cdot 0.6 + 0.9 \cdot 0.4 = 0.45$. (c) $0.09/0.45 = 0.2$.
(d) There is a 0.04 probability of being Canadian and disliking poutine and a 0.55 probability of disliking poutine, so the desired probability is $0.04/0.55 = 4/55$.

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10. (a) $0 + 0.2 + 0.2 = 0.4$. (b) $0.1 + 0.2 = 0.3$. (c) $E(X) = 0.1 \cdot 0 + 0 \cdot 1 + 0.2 \cdot 2 + 0.2 \cdot 3 + 0.5 \cdot 4 = 3$, $E(Y) = 0.4 \cdot 0 + 0.1 \cdot 1 + 0.2 \cdot 2 + 0.1 \cdot 3 + 0.2 \cdot 4 = 1.6$. (d) $E(X + 2Y) = E(X) + 2E(Y) = 6.2$.
(e) $E(X^2) = 0.1 \cdot 0^2 + 0 \cdot 1^2 + 0.2 \cdot 2^2 + 0.2 \cdot 3^2 + 0.5 \cdot 4^2 = 10.6$ so $\text{var}(X) = E(X^2) - [E(X)]^2 = 1.6$, $\sigma(X) = \sqrt{1.6}$.
(f) $E(Y^2) = 0.4 \cdot 0^2 + 0.1 \cdot 1^2 + 0.2 \cdot 2^2 + 0.1 \cdot 3^2 + 0.2 \cdot 4^2 = 5$ so $\text{var}(Y) = E(Y^2) - [E(Y)]^2 = 2.44$, $\sigma(Y) = \sqrt{2.44}$.

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11. (a)

n	0	1	2	3
$P(X = n)$	2/19	15/38	15/38	2/19

(b) $\frac{2}{19} + \frac{15}{38} + \frac{15}{38} = \frac{17}{19}$. (c) $0 \cdot \frac{2}{19} + 1 \cdot \frac{15}{38} + 2 \cdot \frac{15}{38} + 3 \cdot \frac{2}{19} = \frac{3}{2} = 3 \cdot \frac{10}{20}$.
(d) $E(X^2) = 0 \cdot \frac{2}{19} + 1^2 \cdot \frac{15}{38} + 2^2 \cdot \frac{15}{38} + 3^2 \cdot \frac{2}{19} = \frac{111}{38}$ so $\text{var}(X) = E(X^2) - E(X)^2 = \frac{51}{76}$ and $\sigma(X) = \sqrt{\text{var}(X)} = \sqrt{\frac{51}{76}}$.

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12. (a) 0.79 (b) $0.58 - 0.23 = 0.35$ (c) $1 - 0.73 = 0.27$ (d) $E(2X + 5) = 2 \cdot 11 + 5 = 27$ (e) $\sigma(2X + 5) = 2 \cdot 2 = 4$.

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13. (a) 0.1 (b) 0.7 (c) 0.3 (d) 0.2 (e) 0.2,0.5,0.3 for $X = 1, 3, 4$ (f) 0.3,0.4,0.3 for $Y = 3, 4, 5$ (g) 0.8 (h) 0.7 (i) 2.9 and 4.0 (j) 10.9 (k) 1.09 and 1.0440 (l) 0.6 and 0.7746 (m) No (n) 0.4 and 0.4946

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14. (a) 13/24 (b) 11/216 and 0 (c) 3/8 and 5/8 (d) 0 for $x < 0$, $(12x^2 - x^3)/216$ for $0 \le x \le 6$, 1 for $x > 6$ (e) 7/2 and 19 (f) 43/20 and 1.4663

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15. (a) 1/27 (b) 2/27 (c) 16/27 (d) 28/81 (e) 16/81 (f) 0 (g) $(2x + 6)/27$ for $0 \le x \le 3$ (h) $(2y + 1)/6$ for $0 \le y \le 2$ (i) 5/3 and 11/9 (j) 7/2 and 16/9 (k) 2 (l) 13/18 and 0.8498 (m) 23/81 and 0.5329 (n) No (o) $-1/27$ and -0.0818

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16. (a) -1 (b) 7 (c) 24 (d) 9 and 3 (e) 4 and 2 (f) 1 (g) 1/6 (h) 15

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17. (a) Distribution is binomial, $n = 1800$ and $p = 2/3$, so $P(Y = 1200) = \binom{1800}{1200} \cdot (\frac{2}{3})^{1200} \cdot (\frac{1}{3})^{600}$ and $P(1200 \le Y \le 1250) = \binom{1800}{1200} \cdot (\frac{2}{3})^{1200} \cdot (\frac{1}{3})^{600} + \binom{1800}{1202} \cdot (\frac{2}{3})^{1202} \cdot (\frac{1}{3})^{598} + \cdots + \binom{1800}{1250} \cdot (\frac{2}{3})^{1250} \cdot (\frac{1}{3})^{550}$.
(b) $E(Y) = np = 1200$ and $\sigma(Y) = \sqrt{np(1 - p)} = 20$.

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18. (a) Distribution is binomial, $n = 10$ and $p = 0.4$, so $P(\# \ge 2) = 1 - \binom{10}{0} 0.6^{10} - \binom{10}{1} 0.4^1 0.6^9 \approx 0.9476$.
(b) Distribution is binomial, $n = 100$ and $p = 0.8$, so $E(\text{pts}) = np = 80$ and $\sigma(\text{pts}) = \sqrt{np(1 - p)} = 4$.
(c) Expected values and variances add for independent variables. The number of points scored for a shot is scaled by the number of points the shot is worth, so $E(\text{2pt shot}) = 2 \cdot 0.4 = 0.8$, $E(\text{3pt shot}) = 3 \cdot 0.2 = 0.6$, $\text{var}(\text{2pt shot}) = 2^2 \cdot 0.4 \cdot 0.6 = 0.96$, and $\text{var}(\text{3pt shot}) = 3^2 \cdot 0.2 \cdot 0.8 = 1.44$. Then $E(\text{sum}) = 0.8 + 0.6 = 1.4$ and $\text{var}(\text{sum}) = 0.96 + 1.44 = 2.4$, so $\sigma(\text{sum}) = \sqrt{2.4}$.
