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1. A normal distribution X with unknown mean μ_X and standard deviation σ is sampled 5 times, yielding values 6, 11, 12, 9, and 6.

   (a) Find the sample mean and sample standard deviation.

   (b) Find an 80% and a 95% confidence interval for μ_X.

   (c) Test at the 4% significance level whether μ_X is greater than 4.

   (d) Find an 80% and a 95% confidence interval for σ^2.

   (e) Test at the 2% significance level whether σ^2 is less than 100.

   (f) A second normal distribution Y with unknown mean μ_Y but the same standard deviation σ is sampled 4 times, yielding values -1, 8, 12, and 9. Test at the 3% significance level whether the means of X and Y are equal.

   (g) Construct a 50% and a 90% confidence interval for the difference $\mu_X - \mu_Y$ in the sample means.

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2. A bolt manufacturer wants to make sure that the bolts from their factory are within required specifications. They draw a sample of 100 bolts from a lot, obtaining an average diameter of 19.988mm and a sample standard deviation of 0.191mm. It is assumed based on past studies that the bolt sizes are normally distributed. The specifications for this type of bolt require an average diameter of 20.000mm with a standard deviation of at most 0.200mm.

   (a) Test at the 4% significance level whether the average bolt diameter is equal to the specification.

   (b) Construct a 90% and a 99% confidence interval for the average bolt diameter.

   (c) Test at the 4% significance level whether the bolt diameter's standard deviation is within specification.

   (d) Construct a 90% and a 99% confidence interval for the standard deviation in the bolt diameter.

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3. Perform a hypothesis test for each situation: identify the type of test, give the hypotheses, the significance level α, the test statistic and its observed value, the degrees of freedom, the p-value, and the result of the test.
   For two-sample t tests, give the result with both Student's t test (pooled variance) and Welch's t test (unpooled variance).

   (a) Instructors E and D (definitely not the same person) write chapters of notes for their math courses. Instructor E's four chapters total 26pp, 34pp, 18pp, and 50pp, while instructor D's five chapters total 17pp, 20pp, 29pp, 32pp, and 27pp. Assuming that the page totals represent independent samples from a normally distributed population, test at the 7% significance level that E's average chapter length is different from D's average chapter length.

   (b) Researchers want to test whether blaring loud techno music affects students' ability to perform well on exams. Exam scores are expected to be normally distributed. Five students take the exam with loud music and receive scores of 55, 73, 95, 68, and 70. Six students take the exam without loud music and receive scores of 80, 74, 91, 88, 88, and 90. Test at the 1% significance level whether students performed worse with loud music than without loud music.

   (c) Researchers want to test whether the presence of adorable kittens improves students' ability to pay attention during boring lectures. Eight students are monitored during one lecture with kittens and another lecture without kittens to determine whether there is a difference in the number of times they lose focus; the results are below. Test at the 6% significance level whether the number of times they lost focus was changed by the presence of kittens.

Kittens \ Student	D	E	F	J	K1	K2	L	M
With Kittens	3	18	14	9	21	9	11	5
Without Kittens	3	8	13	11	17	16	8	6

(d) The sizes of three randomly-chosen offices in the math department are 135sqft, 148sqft, and 120sqft, while the sizes of four randomly-chosen offices in the business department are 180sqft, 192sqft, and 173sqft. Assuming the office sizes are approximately normally distributed, test at the 4% significance levels whether the offices in the business department are larger than the offices in the math department.

(e) At a certain sporting event, various countries were represented by both junior and senior participants. Test at the 5% significance level whether the seniority level was independent of nationality.

	USA	UK	China	Russia
Senior	46	27	26	19
Junior	75	40	44	37

(f) The faculty salaries at a university are approximately normally distributed. The university samples 30 male faculty and finds their average salary to be \$108591 with a sample standard deviation of \$21291, while a sample of 18 female faculty has average salary \$91513 with a sample standard deviation of \$23515. Test at the 3% significance level whether the female faculty are paid less than the male faculty.
Also, find 80% and 95% confidence intervals for the difference in the two populations' salaries.

(g) Researchers want to test whether an applicant's race is independent of their chances of receiving a callback on an interview[1]. They send 2,425 resumes with White-sounding names and receive 234 callbacks and 2191 rejections, and also 2,425 resumes with Black-sounding names and receive 157 callbacks and 2268 rejections. (The resumes are otherwise identical.) Test at the 1% significance level whether the perceived race of the applicant was independent from whether the applicant received a callback.

(h) Meta-researchers want to test whether there is evidence that p-hacking has occurred in published papers. They tabulate published p-values from papers that either fall "just under" a common significance threshold or "just over" a significance threshold. If no p-hacking has occurred, for p-values in each cluster, half should be "just over" and the other half should be "just under". Test at the 0.1% significance level whether the data conform to the predicted model.

p-value	0.001	0.01	0.05
Just Over	94	76	87
Just Under	114	120	279

(i) The list prices for four statistics textbooks are \$193.95, \$171.89, \$221.80, and \$215.32. The list prices for three calculus textbooks are \$219.99, \$182.38, and \$187.70. Assume that textbook prices are approximately normally distributed. Test at the 6% significance level whether the average prices for statistics textbooks are higher than for calculus textbooks.
Also, find 50% and 80% confidence intervals for the difference in the average book prices.

(j) An investor wants to determine whether a particular mutual fund is a good investment and also a stable investment, so they measure the yearly rate of return over four different years, obtaining 5.1%, 4.5%, 7.8%, and 4.6% returns. Test at the 3% significance level whether the average rate of return is above 4%, and also test at the 4% significance level whether the standard deviation in the rate of return is below 2.5%.
Also, find 95% confidence intervals for the average and standard deviation.

(k) The unreliable pollster from the last course lecture gets a new job doing accounting. Their work is once again suspected of being fraudulent, so they submit a supposedly random sample of 1000 units digits from a wide range of calculations. Test at the 0.1% significance level whether (i) the digits appear to be uniformly distributed, and (ii) whether these results are suspiciously accurate.

Units Digits	0	1	2	3	4	5	6	7	8	9
Observed	100	101	98	99	102	98	99	97	103	103

(l) The ex-pollster ex-accountant gets a job recording geographic data. Yet again, their work is suspected of being fraudulent, so a random sample of 1000 leading digits from a wide range of calculations are tabulated. Test at the 0.1% significance level whether (i) the digits appear to be Benford-distributed, and (ii) whether these results are suspiciously accurate.

Leading Digits	1	2	3	4	5	6	7	8	9
Observed	328	161	120	106	77	47	66	56	39
Benford (%)	30.1%	17.6%	12.5%	9.7%	7.9%	6.7%	5.8%	5.1%	4.6%

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[1] Bertrand and Mullainathan, "Are Emily and Greg More Employable Than Lakisha and Jamal? A Field Experiment on Labor Market Discrimination", Amer. Econ. Rev. (2004), https://cos.gatech.edu/facultyres/Diversity_Studies/Bertrand_LakishaJamal.pdf .