1. For each independent sample of the given random variable, find (i) the likelihood function $L(\theta)$, (ii) the derivative of the log-likelihood $\frac{d}{d\theta}[\ln L(\theta)]$, and (iii) the maximum likelihood estimate $\hat{\theta}$:
   (a) $p_X(n;\theta) = \theta(1-\theta)^n$ for integers $n \geq 0$, with sample values 3, 4, 1, and 6.
   (b) $p_X(n;\theta) = \theta^n e^{-\theta}/n!$ for integers $n \geq 0$, with sample values 5, 3, and 6.
   (c) $p_X(x;\theta) = \frac{1}{\theta}e^{-x/\theta}$ for $x \geq 0$, with sample values 1, 2, and 5.
   (d) $p_X(x;\theta) = e^{-(x-\theta)^2}/\sqrt{\pi}$ for all real x, with sample values 1, -2, 0.
   (e) $p_X(x;\theta) = 2(\theta - x)/\theta^2$ for $0 \leq x \leq \theta$, with sample values 2 and 5. (Note $\theta > 5$ here.)

2. A Poisson distribution with parameter λ is independently sampled four times, yielding values x_1, x_2, x_3, x_4. For each estimator (i) find its expected value and decide whether it is biased and (ii) find its variance. Then decide which of the unbiased estimators is the most efficient.
   (a) $\hat{\lambda}_1 = \frac{1}{2}(x_1 + x_3)$. (b) $\hat{\lambda}_2 = \frac{1}{4}(x_1+x_2+x_3+x_4)$ (c) $\hat{\lambda}_3 = x_1 - x_2 + 2x_3$. (d) $\hat{\lambda}_4 = \frac{1}{5}(x_1+x_2+2x_3+x_4)$

3. A normal distribution with mean μ and variance 4 is sampled once, yielding the value x. A normal distribution with mean 3μ and variance 3 is sampled once, yielding y. Consider estimators for μ of the form $\hat{\mu} = ax + by$.
   (a) Find $E(\hat{\mu})$ and $\text{var}(\hat{\mu})$. (b) Find the values of a and b that give the most efficient unbiased estimator $\hat{\mu}$.

4. A normal distribution with unknown mean μ is sampled five times, yielding values 0, -2, 3, 1, 6.
   (a) If the standard deviation is known to be 4, find 80%, 90%, 95%, and 99.5% CIs for μ.
   (b) If the standard deviation is known to be 4, test at the 10%, 3%, and 1% significance levels the hypothesis that the mean is equal to 0.
   (c) If the standard deviation is unknown, find 80%, 90%, 95%, and 99.5% CIs for μ.

5. Researchers want to test whether hearing a certain type of music improves the intelligence of infants. 880 infants are randomly assigned to receive a treatment: 330 hear classical music, 350 hear thrash metal, and 200 hear a placebo (nature sounds) for two months. The infants are then assessed on a basic skills scale whose scores are normally distributed with standard deviation 1. The classical music group has an average score of 6.81, the thrash metal group has an average score of 6.90, and the placebo group has an average score of 6.83.
   (a) Test at the 4% level whether classical music improves the skills score relative to a placebo.
   (b) Test at the 4% level whether thrash metal improves the skills score relative to a placebo.
   (c) Test at the 4% level whether classical music and thrash metal have different skills scores.
   (d) If there is actually no difference among any of the three groups, what is the approximate probability of making a type I error in *at least one* of the hypothesis tests (a)-(c)?
   (e) Briefly explain why it would be improper statistical practice to report the result of only the tests performed in (a)-(c) that have a p-value below the 4% threshold.

6. The math department wants to determine whether students actually enjoy Math 3081.[1] To determine this, they randomly poll 50 students in the course and find that 38% (19 students) enjoy it.
   (a) Find 80%, 90%, 95%, and 99% confidence intervals for the true proportion of students who enjoy the course.
   (b) If the department wanted to have confidence intervals that were 1/3 as wide in part (a), how many students would they need to poll?
   (c) Assuming the sample above is representative, if the department wanted a 95% margin of error of $\pm 2\%$, how many students would they need to poll? How would the answer change if the department had no idea of the true proportion of students who enjoy the course?
   (d) Test at the 11%, 3%, and 0.5% significance levels that exactly 30% of the students enjoy the course.
   (e) Test at the 11%, 3%, and 0.5% significance levels that less than half of students enjoy the course.
   (f) For each hypothesis test performed in (d)-(e), identify whether it would be a type I error, type II error, or correct decision if (i) the null hypothesis were true, or (ii) the true proportion was equal to 45%.

[1] As noted on the last review sheet, the actual proportion of such students is 0%.

7. Perform a hypothesis test for each situation: give the hypotheses, the significance level α, the test statistic and its observed value, the p-value, and the result of the test.
   (a) Researchers want to test whether crystal healing treatments are more effective than a placebo. 134 patients are randomly assigned to receive a treatment: 67 receive crystal healing while the other 67 receive an equivalent treatment with regular rocks. 22 of the crystal healing patients and 20 of the placebo patients report improved wellness. Test at the 4% level whether crystal healing was more effective than a placebo.
   (b) The tenure-track faculty salaries at a university are approximately normally distributed with standard deviation \$30000. The university samples 100 male faculty and finds their average salary to be \$108591, while a sample of 25 female faculty finds their average salary to be \$91513. Test at the 3% significance level that the female faculty are paid less than the male faculty.
   Also, find 95% confidence intervals for the two populations' salaries.
   (c) Researchers want to test whether people with Black-sounding names are less likely to receive job interviews than people with White-sounding names[2]. They send 2,425 resumes with White-sounding names and receive 234 callbacks, and also 2,425 resumes with Black-sounding names and receive 157 callbacks. (The resumes are otherwise identical.) Test at the 0.1% significance level that the resumes with Black-sounding names were less likely to receive an interview than ones with a White-sounding name.
   Also, find 95% confidence intervals for the two callback percentages.

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8. Researchers want to test whether taking hydroxychloroquine lowers the rate of hospitalization in patients with COVID-19.[3] It is estimated that patients who do not take the drug require hospitalization within 15 days 15.6% of the time. 30 patients out of 159 who took hydroxychloroquine required hospitalization within 15 days.
   (a) Test whether hydroxychloroquine affects the hospitalization rate at the 20%, 9%, and 2% significance levels.
   (b) Test whether hydroxychloroquine lowers the hospitalization rate at the 20%, 9%, and 2% significance levels.
   (c) Test whether hydroxychloroquine increases the hospitalization rate at the 20%, 9%, and 2% levels.
   (d) Based on the results of (a)-(c), critique the accuracy of the following statements: (i) "the study proves that hydroxychloroquine is effective", (ii) "the study proves that hydroxychloroquine is not effective", (iii) "it is impossible to conclude from the study whether hydroxychloroquine is effective".
   (e) If the number of patients were tripled (but all the percentages stayed the same), describe the effect on the power of the test. Would any of the answers in the previous parts change?

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9. Find 50%, 80%, 90%, and 99% confidence intervals for each requested statistic based only on the given information (assume that all of the statistics are approximately normally distributed):
   (a) The online list prices of four randomly-chosen possible statistics textbooks for Math 3081 are \$193.95, \$171.89, \$221.80, and \$215.32. Find confidence intervals for the average price of the statistics textbooks.
   (b) The scores of ten randomly-chosen students on a Math 3081 exam are 80, 77, 73, 68, 71, 72, 77, 74, 75, 75. Find confidence intervals for the average score on the exam.
   (c) The sizes of three randomly-chosen offices in the math department are 135sqft, 148sqft, and 120sqft. Find confidence intervals for the average math department office size.
   (d) The annual salaries of three randomly-chosen Northeastern faculty members are \$104000, \$135000, and \$89000. Find confidence intervals for the average faculty salary.
   (e) The annual salaries of four randomly-chosen Northeastern senior administrators are \$290000, \$165000, \$184000, and \$201000. Find confidence intervals for the average senior administrator salary.
   (f) The annual salaries of five randomly-chosen Northeastern staff members are \$51000, \$48000, \$35000, \$24000, and \$77000. Find confidence intervals for the average staff salary.

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[2] Bertrand and Mullainathan, "Are Emily and Greg More Employable Than Lakisha and Jamal? A Field Experiment on Labor Market Discrimination", Amer. Econ. Rev. (2004), https://cos.gatech.edu/facultyres/Diversity_Studies/Bertrand_LakishaJamal.pdf .

[3] Cavalcanti *et al.*, Hydroxychloroquine with or without Azithromycin in Mild-to-Moderate Covid-19, NEJM (July 2020), https://www.nejm.org/doi/full/10.1056/NEJMoa2019014 .