1. (a) (i) $\theta^4(1-\theta)^{14}$ (ii) $\frac{4}{\theta} - \frac{14}{1-\theta}$ (iii) $\hat{\theta} = 4/18$.
   (b) (i) $\theta^{14}e^{-3\theta}/518400$ (ii) $\frac{14}{\theta} - 3$ (iii) $\hat{\theta} = 14/3$.
   (c) (i) $\theta^{-3}e^{-8/\theta}$ (ii) $\frac{-3}{\theta} + \frac{8}{\theta^2}$ (iii) $\hat{\theta} = 8/3$.
   (d) (i) $e^{-(1-\theta)^2-(-2-\theta)^2-(0-\theta)^2}\pi^{-3/2}$ (ii) $2(1-\theta)+2(-2-\theta)+2(0-\theta)$ (iii) $\hat{\theta} = -1/3$.
   (e) (i) $4(\theta-2)(\theta-5)/\theta^4$ (ii) $\frac{1}{\theta-2} + \frac{1}{\theta-5} - \frac{4}{\theta}$ (iii) $\hat{\theta} = 8$ (there is a second root $\hat{\theta} = 5/2$ but it is less than 5).

---

2. (a) $E(\hat{\lambda}_1) = \frac{1}{2}[E(x_1)+E(x_2)] = \lambda$, unbiased, $\mathrm{var}(\hat{\lambda}_1) = \frac{1}{4}[\mathrm{var}(x_1)+\mathrm{var}(x_2)] = \lambda/2$.
   (b) $E(\hat{\lambda}_2) = \frac{1}{4}[E(x_1)+E(x_2)+E(x_3)+E(x_4)] = \lambda$, unbiased
   $\mathrm{var}(\hat{\lambda}_2) = \frac{1}{16}[\mathrm{var}(x_1)+\mathrm{var}(x_2)+\mathrm{var}(x_3)+\mathrm{var}(x_4)] = \lambda/4$.
   (c) $E(\hat{\lambda}_3) = E(x_1) - E(x_2) + 2E(x_3) = 2\lambda$, biased
   $\mathrm{var}(\hat{\lambda}_3) = \mathrm{var}(x_1)+\mathrm{var}(x_2)+4\mathrm{var}(x_3) = 6\lambda$.
   (d) $E(\hat{\lambda}_4) = \frac{1}{5}[E(x_1)+E(x_2)+2E(x_3)+E(x_4)] = \lambda$, unbiased,
   $\mathrm{var}(\hat{\lambda}_4) = \frac{1}{25}[\mathrm{var}(x_1)+\mathrm{var}(x_2)+4\mathrm{var}(x_3)+\mathrm{var}(x_4)] = 7\lambda/25$.
   (e) The estimator $\hat{\lambda}_2$ is the most efficient since its variance is the smallest.

---

3. (a) By expected value properties, $E(\hat{\mu}) = E(ax)+E(by) = aE(x)+bE(y) = (a+3b)\mu$,
   By variance properties, $\mathrm{var}(\hat{\mu}) = \mathrm{var}(ax)+\mathrm{var}(by) = a^2\mathrm{var}(x)+b^2\mathrm{var}(y) = 4a^2+3b^2$.
   (b) Need $a+3b=1$ for unbiased, so $a = 1-3b$. Then $\mathrm{var}(\hat{\mu}) = 4(1-3b)^2+3b^2 = 39b^2-24b+4$ has derivative $78b-24$ which is zero for $b = 4/13$, which is a min. Therefore $a = 1/13$, $b = 4/13$ gives the minimum.

---

4. (a) $\mu = 1.6$, $\sigma = 4$, 80%: $(-0.6926, 3.8926)$, 90%: $-1.3425, 4.5425)$, 95%: $(-1.9062, 5.1062)$, 99.5: $(-3.4213, 6.6213)$.
   (b) p-value is $2P(N_{0,4/\sqrt{5}} > 8/5) = 0.3711$, fail to reject at 10%, 3%, 1%.
   (c) $\mu = 1.6$, $S = 3.0496$, 80%: $(-0.4910, 3.6910)$, 90%: $(-1.3075, 4.5075)$, 95%: $(-2.1866, 5.3866)$, 99.5: $(-6.0341, 9.2341)$.

---

5. (a) H_0: $\mu_c = \mu_p$, H_a: $\mu_c > \mu_p$, $\sigma = \sqrt{\frac{1}{330}+\frac{1}{200}} = 0.0896$, p-value is $P(N_{0,0.0896} > -0.02) = 0.5883$, fail to reject H_0.
   Alternatively, with H_0: $\mu_c = \mu_p$, H_a: $\mu_c < \mu_p$, the p-value is $P(N_{0,0.0896} < -0.02) = 0.4117$, fail to reject H_0.
   (b) H_0: $\mu_t = \mu_p$, H_a: $\mu_t > \mu_p$, $\sigma = \sqrt{\frac{1}{350}+\frac{1}{200}} = 0.0886$, p-value is $P(N_{0,0.0886} > 0.07) = 0.2147$, fail to reject H_0.
   (c) H_0 :$\mu_c = \mu_t$, H_a : $\mu_c \neq \mu_t$, $\sigma = \sqrt{\frac{1}{350}+\frac{1}{330}} = 0.0767$, p-value is $2P(N_{0,0.0767} > 0.09) = 0.2406$, fail to reject H_0.
   (d) The probability of a type I error in each case is 4%, so the probability of making at least one error is $1-0.96^3 \approx 11.5\%$, assuming independence. (A reasonable estimate is also $4\%+4\%+4\% = 12\%$.)
   (e) Publishing only statistically-significant outcomes creates a bias in the literature, especially since performing multiple tests increases the probability of a false-positive result, as calculated in (d). All tests performed should always be reported, and corrections for multiple tests must be included.

---

6. (a) $\mu = 0.38$, $\sigma = 0.0686$, 80%: $(0.2920, 0.4680)$, 90%: $(0.2671, 0.4929)$, 95%: $(0.2455, 0.5145)$, 99%: $(0.2032, 0.5568)$.
   (b) 9 times as many students: 450 in total.
   (c) Need $n = \frac{\hat{p}(1-\hat{p})}{(0.02/1.9600)^2} \approx 2262.7$ (so 2263). If $\hat{p}$ is unknown then worst case is $\hat{p} = 0.5$ with $n = 2401$.
   (d) H_0: $p = 0.3$, H_a: $p \neq 0.3$, p-value is $P(|B_{50,0.3} - 15| \geq |19-15|) \approx 2P(N_{15,3.2404} > 18.5) = 0.2801$, fail to reject H_0.

(e) H_0: $p = 0.5$, H_a: $p < 0.5$, p-value is $P(B_{50,0.5} < 19) \approx P(N_{25,3.5355} < 19.5) = 0.0599$, reject / fail to reject / fail to reject H_0.

(f) (d-i) correct / correct / correct, (d-ii) type II / type II / type II,
(e-i) type I / correct / correct, (e-ii) correct / type II / type II.

---

7. (a) H_0: $p_c = p_p$, H_a: $p_c > p_p$, $p_{\text{pool}} = 0.3134$, $\sigma_{\text{pool}} = 0.08015$, $\hat{p}_c - \hat{p}_p = 0.02985$, p-value is $P(N_{0,0.08015} > 0.02985) = 0.3548$, fail to reject H_0.

   (b) H_0: $\mu_m = \mu_f$, H_a: $\mu_m > \mu_f$, $\mu_{m-f} = 17078$, $\sigma_{\text{pool}} = 6708$, p-value is $P(N_{0,6708} > 17078) = 0.0054$, reject H_0.
   95% CI for male: $(102711, 114471)$, 95% CI for female: $(79753, 103273)$. Note $\sigma_{\text{male,avg}} = 30000/\sqrt{100} = 3000$ and $\sigma_{\text{female,avg}} = 30000/\sqrt{25} = 6000$.

   (c) H_0: $p_w = p_b$, H_a: $p_w > p_b$, $p_{\text{pool}} = 0.0808$, $\sigma_{\text{pool}} = 0.00783$, p-value is $P(N_{0,0.00783} > 0.03176) = 2.5 \cdot 10^{-5}$, reject H_0.
   95% CI for White: $(8.47\%, 10.82\%)$, 95% CI for Black: $(5.49\%, 7.45\%)$. Note $\hat{p}_{\text{White}} = 9.69\%$, $\sigma_{\text{White,prop}} = 0.71\%$, $\hat{p}_{\text{Black}} = 6.47\%$, $\sigma_{\text{Black,prop}} = 0.50\%$.

---

8. (a) H_0: $p = 0.156$, H_a: $p \neq 0.156$, $np = 24.8$, $\sqrt{np(1-p)} = 4.575$, p-value is $\approx 2P(N_{24.8,4.575} > 29.5) = 0.3043$, fail to reject H_0.

   (b) H_0: $p = 0.156$, H_a: $p < 0.156$, $np = 24.8$, $\sqrt{np(1-p)} = 4.575$, p-value is $\approx P(N_{24.8,4.575} < 29.5) = 0.8479$, fail to reject H_0.

   (c) H_0: $p = 0.156$, H_a: $p > 0.156$, $np = 24.8$, $\sqrt{np(1-p)} = 4.575$, p-value is $\approx P(N_{24.8,4.575} > 29.5) = 0.1521$, reject H_0 / fail to reject H_0 / fail to reject H_0.

   (d) (i) One study cannot prove anything definitively. It also provides essentially zero evidence toward the hypothesis that hydroxychloroquine is effective in lowering the hospitalization rate, from test (b).
   (ii) One study cannot prove anything definitively. It provides somewhat weak evidence toward the hypothesis that hydroxychloroquine increases the hospitalization rate, from test (c).
   (iii) This is also not entirely accurate, because test (c) does provide weak evidence suggesting that hydroxychloroquine increases the hospitalization rate.

   (e) The power of the test, and hence the strength of the conclusions, increases with a larger sample size. The p-values here would shift to 0.0752, 0.9624, 0.0376.

---

9. These are all t confidence intervals. Use the t-table with $n-1$ degrees of freedom to get $t_{\alpha/2,n}$, measuring the number of standard deviations in the margin of error.

   (a) $\mu = 200.74$, $S = 22.6166$, 50%: $(192.09, 209.39)$, 80%: $(182.22, 219.26)$, 90%: $(174.13, 227.35)$, 99%: $(134.69, 266.79)$.

   (b) $\mu = 74.2$, $S = 3.4254$, 50%: $(73.44, 74.96)$, 80%: $(72.70, 75.70)$, 90%: $(72.21, 76.19)$, 99%: $(70.68, 77.72)$.

   (c) $\mu = 134.33$, $S = 14.0119$, 50%: $(127.73, 140.94)$, 80%: $(119.08, 149.59)$, 90%: $(110.71, 157.96)$, 99%: $(54.04, 214.62)$.

   (d) $\mu = 109333$, $S = 23459$, 50%: $(98000, 120000)$, 80%: $(84000, 135000)$, 90%: $(70000, 149000)$, 99%: $(-25000, 244000)$.

   (e) $\mu = 201000$, $S = 55323$, 50%: $(189000, 231000)$, 80%: $(165000, 255000)$, 90%: $(145000, 275000)$, 99%: $(48000, 372000)$.

   (f) $\mu = 47000$, $S = 19937$, 50%: $(40000, 54000)$, 80%: $(33000, 61000)$, 90%: $(28000, 66000)$, 99%: $(6000, 88000)$.
