1. Given that discrete random variables X and Y have the joint distribution table given below, find:

$X \backslash Y$	3	4	5
1	0.1	0.1	0
3	0.2	0.2	0.1
4	0	0.1	0.2

   (a) $P(X = 1, Y = 4)$.
   (b) $P(Y > X)$.
   (c) $P(X = Y)$.
   (d) $P(X + Y = 7)$.
   (e) The marginal distribution of X
   (f) The marginal distribution of Y
   (g) $P(X > 1)$.
   (h) $P(Y < 5)$.
   (i) $E(X)$ and $E(Y)$.
   (j) $E(X + 2Y)$.
   (k) $\text{var}(X)$ and $\sigma(X)$.
   (l) $\text{var}(Y)$ and $\sigma(Y)$.
   (m) If X, Y are independent.
   (n) $\text{cov}(X, Y)$ and $\text{corr}(X, Y)$.

2. The continuous random variable Z has pdf $p(x) = (8x - x^2)/72$ for $0 \le x \le 6$ and 0 for other x. Find:
   (a) $P(1 \le Z \le 4)$.
   (b) $P(Z < 1)$ and $P(Z = 1)$.
   (c) $P(Z < 3)$ and $P(Z > 3)$.
   (d) The c.d.f. (cumulative) for Z.
   (e) $E(Z)$ and $E(4Z + 5)$.
   (f) $\text{var}(Z)$ and $\sigma(Z)$.

3. If the continuous random variables X and Y have joint pdf $p(x, y) = c \cdot (x + 3y)$ for $0 \le x \le 3$, $0 \le y \le 2$, find
   (a) The value of c.
   (b) $P(0 \le X \le 1, 0 \le Y \le 1)$.
   (c) $P(X < 2)$.
   (d) $P(X < Y)$.
   (e) $P(X + Y < 2)$.
   (f) $P(X = 1)$.
   (g) The marginal distribution of X
   (h) The marginal distribution of Y
   (i) $E(X)$ and $E(Y)$.
   (j) $E(X^2)$ and $E(Y^2)$.
   (k) $E(XY)$.
   (l) $\text{var}(X)$ and $\sigma(X)$.
   (m) $\text{var}(Y)$ and $\sigma(Y)$.
   (n) If X, Y are independent.
   (o) $\text{cov}(X, Y)$ and $\text{corr}(X, Y)$.

4. Suppose $E(X) = 1$, $E(Y) = 3$, $E(X^2) = 10$, $E(Y^2) = 13$, and $E(XY) = 4$. Find:
   (a) $E(2X - 3)$.
   (b) $E(X + 2Y)$.
   (c) $E(XY + 2X^2)$.
   (d) $\text{var}(X)$ and $\sigma(X)$.
   (e) $\text{var}(Y)$ and $\sigma(Y)$.
   (f) $\text{cov}(X, Y)$.
   (g) $\text{corr}(X, Y)$.
   (h) $\text{var}(X + Y)$.

5. The continuous random variable X is normally distributed with $\mu = 10$ and $\sigma = 2$. Using the values below, find:

z	-1	0	0.5	1	1.5	2	2.5	3
$P(N_{0,1} \le z)$	0.1587	0.5	0.6915	0.8413	0.9332	0.9772	0.9938	0.9987

   (a) $P(X < 14)$.
   (b) $P(8 \le X \le 12)$
   (c) $P(X > 13)$.
   (d) $E(2X + 5)$.
   (e) $\sigma(2X + 5)$.

6. A coin with probability 2/3 of landing heads is flipped 1800 times. Let Y be the random variable counting the total number of heads.
   (a) Find exact expressions for $P(Y = 1200)$ and $P(1200 \le Y \le 1250)$ (you need not evaluate them).
   (b) Find $E(Y)$ and $\sigma(Y)$.
   (c) Use the normal approximation with continuity correction to estimate $P(1200 \le Y \le 1250)$.

7. A basketball player has a 0.8 probability of making a 1-point free throw, a 0.4 probability of making a 2-point shot, and a 0.2 probability of making a 3-point shot. All shots are independently likely to score.
   (a) If the player takes 10 2-point shots, what is the probability she scores on at least 2 shots?
   (b) Find the expected number, and standard deviation, of total points from 100 free throws.
   (c) Find the expected number, and standard deviation, of total points from a 3-point shot plus a 2-point shot.
   (d) If the player takes 1000 free throws, 2000 2-point shots, and 500 3-point shots, describe the approximate distribution of the total number of points she scores.

8. The weights of widgets are normally distributed with mean 10.05g and standard deviation 0.10g. Sample A consists of 10 widgets and sample B consists of 25 widgets.
   (a) Describe the distributions of the respective average weights of Sample A and Sample B.
   (b) Find the probability that a random widget has weight over 10.15g.
   (c) Find the probability that at least one widget in Sample A has weight over 10.15g.
   (d) Find the mean and standard deviation of the total weight of Sample A.
   (e) Find the probability that the total weight of Sample A exceeds 101g.
   (f) Find the probability that the average weight of Sample A is less than 9.95g.
   (g) Describe the distribution of the difference in the average weights of Sample A and Sample B.
   (h) Find the probability that the average weight of Sample A exceeds the average of B by 0.05g or more.

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9. You wait for the Orange Line at Ruggles during rush hour: your wait time is uniformly distributed between 0min and 7min.
   (a) What is the probability that you will have to wait 5+ minutes (5 minutes or more)?
   (b) Find the expected value and standard deviation of your wait time.
   (c) If you take the train 75 times, describe the approximate distribution of your average wait time.
   (d) If you take the train 75 times, estimate the probability that your average wait time exceeds 3.57 minutes.
   (e) If you take the train 75 times, estimate the probability that you have to wait 5+ minutes at least 30 times.

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10. On average, the Green Line through Northeastern has a service interruption 3 times per week.
    (a) If X is the random variable measuring the number of times there is a service interruption in one week, what type of distribution is X, and what is its pdf?
    (b) What is the probability that there is no service interruption this week?
    (c) What is the probability that there are at least 5 service interruptions this week?
    (d) What is the probability that there is at least 1 service interruption today?
    (e) What is the probability that there are exactly 15 service interruptions within the next 30 days?

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11. You wait for the #28 bus at Ruggles during a snowstorm. Your expected wait time is 30 minutes, independent of the amount of time you have already been waiting.
    (a) If Y is the random variable measuring your wait time, what type of distribution is Y, and what is its pdf?
    (b) Find the probability that you wait at least 30 minutes.
    (c) Find the probability that the bus comes within the next 10 minutes.

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12. Students in Math 3081 really enjoy the course about 2% of the time[1]. Assume there are 130 students in the course.
    (a) Find the exact probability that either 0 or 1 student really enjoys the course.
    (b) An approximation to the probability in (a) can be found using a Poisson model. What is the parameter for this model, and what is the probability estimate?
    (c) An approximation to the probability in (a) can also be found using a normal model. What are the parameters for this model, and what is the probability estimate?
    (d) Which estimate (Poisson or normal) is more accurate? Briefly explain why you should expect this to be the case.

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[1] This is false; the actual value is 0%.