1. (a) $88 - 54 = 34$. (b) $130 - (34 + 73) = 23$.

2. (a) There are 7 choices for letters 3,4,5 so $1 \cdot 1 \cdot 7 \cdot 7 \cdot 7 = 343$ strings.
(b) There are 7 choices for first letter, 6 for second, etc, so $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$ strings.
(c) There are 5 choices for each letter, so there are $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3125$ strings.
(d) There are 7^5 total strings and this is the complement of (b), so $7^5 - 2520 = 14287$ strings.
(e) There are $6^5 = 7776$ strings without a B, so there are $7^5 - 6^5 = 9031$ strings.
(f) There are 7 choices for first letter and 6 for each other letter, so $7 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 9072$ strings.

3. (a) $\binom{14}{2} \cdot \binom{12}{2} \cdot \binom{10}{2} = 270270$ possible choices. (b) $\frac{1}{3!} \cdot \binom{14}{2} \cdot \binom{12}{2} \cdot \binom{10}{2} = 45045$ possible choices.

4. (a) $1/2^{10} = 1/1024$.
(b) $1/4$.
(c) $\binom{10}{4}/2^{10} = 105/512$.
(d) $\frac{1}{2^{10}}[\binom{10}{8} + \binom{10}{9} + \binom{10}{10}] = 7/128$.
(e) $1/4$.
(f) $\frac{1}{2^{10}}[\binom{10}{6} + \binom{10}{7} + \cdots + \binom{10}{10}] = 193/512$.

5. (a) $6/216 = 1/36$.
(b) $6 \cdot 5 \cdot 3/216 = 5/12$.
(c) $6 \cdot 5 \cdot 4/216 = 5/9$.
(d) $3!/216 = 1/36$.
(e) $4 \cdot 4 \cdot 4/216 = 8/27$.
(f) $1 - \frac{5 \cdot 5 \cdot 5}{216} = 91/216$.
(g) $\frac{1}{216}/\frac{8}{27} = 1/64 = 1/4^3$.
(h) $\frac{3}{216}/\frac{5}{12} = 1/30 = 1/(6 \cdot 5)$.
(i) $\frac{1}{36}/\frac{91}{216} = 6/91$.
(j) $\frac{1}{36}/\frac{5}{9} = 1/20 = 1/\binom{6}{3}$.

6. (a) $\frac{10}{18} \cdot \frac{9}{17} \cdot \frac{8}{16} \cdot \frac{7}{15} = \frac{7}{102}$.
(b) $4 \cdot \frac{10}{18} \cdot \frac{8}{17} \cdot \frac{7}{16} \cdot \frac{6}{15} = \frac{28}{153}$.
(c) $P(\geq 1 \text{ red}) = 1 - P(\text{none red}) = 1 - \frac{8}{18} \cdot \frac{7}{17} \cdot \frac{6}{16} \cdot \frac{5}{15} = \frac{299}{306}$, and the intersection of these events is drawing 4 reds, so the desired conditional probability is $\frac{7}{102}/\frac{299}{306} = \frac{21}{299}$.
(d) $P(\geq 3 \text{ red}) = \frac{10}{18} \cdot \frac{9}{17} \cdot \frac{8}{16} \cdot \frac{7}{15} + 4 \cdot \frac{8}{18} \cdot \frac{10}{17} \cdot \frac{9}{16} \cdot \frac{8}{15} = \frac{13}{34}$, and the intersection of these events is drawing orange-red-red-red of probability $\frac{8}{18} \cdot \frac{10}{17} \cdot \frac{9}{16} \cdot \frac{8}{15} = \frac{4}{51}$, so the desired conditional probability is $\frac{4}{51}/\frac{13}{34} = \frac{8}{39}$.

7. (a) $1 - P(A) = 0.6$.
(b) $P(B|A) \cdot P(A) = 0.32$.
(c) $P(B|A^c) \cdot P(A^c) = 0.06$.
(d) $P(A \cap B) + P(A \cap B^c) = 0.38$.
(e) $1 - P(B) = 0.62$.
(f) $P(A) + P(B) - P(A \cap B) = 0.46$.
(g) $P(A) - P(A \cap B) = 0.08$.
(h) $P(A \cap B)/P(B) = 16/19$.
(i) $P(A \cap B^c)/P(A) = 1/5$.
(j) $1 - P(A \cup B) = 0.54$.
(k) $P(A^c \cap B^c)/P(B^c) = 27/31$.
(l) $1 - P(A^c \cap B) = 0.94$.

8. (a) Mutually disjoint means $P(A \cap B) = 0$, so then $P(A \cup B) = 0.3 + 0.4 - 0 = 0.7$.
(b) Independent means $P(A \cap B) = P(A)P(B) = 0.12$, so then $P(A \cup B) = 0.3 + 0.4 - 0.12 = 0.58$.

9. (a) $0.15 \cdot 0.6 = 0.09$. (b) $0.15 \cdot 0.6 + 0.9 \cdot 0.4 = 0.45$. (c) $0.09/0.45 = 0.2$.
(d) 0.04 prob. of being Canadian and disliking poutine, 0.55 prob. of disliking poutine, so $0.04/0.55 = 4/55$.

10. (a) $0 + 0.2 + 0.2 = 0.4$. (b) $0.1 + 0.2 = 0.3$. (c) $E(X) = 0.1 \cdot 0 + 0 \cdot 1 + 0.2 \cdot 2 + 0.2 \cdot 3 + 0.5 \cdot 4 = 3$, $E(Y) = 0.4 \cdot 0 + 0.1 \cdot 1 + 0.2 \cdot 2 + 0.1 \cdot 3 + 0.2 \cdot 4 = 1.6$. (d) $E(X + 2Y) = E(X) + 2E(Y) = 6.2$.
(e) $E(X^2) = 0.1 \cdot 0^2 + 0 \cdot 1^2 + 0.2 \cdot 2^2 + 0.2 \cdot 3^2 + 0.5 \cdot 4^2 = 10.6$ so $\text{var}(X) = E(X^2) - [E(X)]^2 = 1.6$, $\sigma(X) = \sqrt{1.6}$.
(f) $E(Y^2) = 0.4 \cdot 0^2 + 0.1 \cdot 1^2 + 0.2 \cdot 2^2 + 0.1 \cdot 3^2 + 0.2 \cdot 4^2 = 5$ so $\text{var}(Y) = E(Y^2) - [E(Y)]^2 = 2.44$, $\sigma(Y) = \sqrt{2.44}$.

11. (a)

n	0	1	2	3
$P(X = n)$	2/19	15/38	15/38	2/19

(b) $\frac{2}{19} + \frac{15}{38} + \frac{15}{38} = \frac{17}{19}$. (c) $0 \cdot \frac{2}{19} + 1 \cdot \frac{15}{38} + 2 \cdot \frac{15}{38} + 3 \cdot \frac{2}{19} = \frac{3}{2} = 3 \cdot \frac{10}{20}$.
(d) $E(X^2) = 0 \cdot \frac{2}{19} + 1^2 \cdot \frac{15}{38} + 2^2 \cdot \frac{15}{38} + 3^2 \cdot \frac{2}{19} = \frac{111}{38}$ so $\text{var}(X) = E(X^2) - E(X)^2 = \frac{51}{76}$ and $\sigma(X) = \sqrt{\text{var}(X)} = \sqrt{\frac{51}{76}}$.