



**Problem 2.** (10 pts)

A box contains 3 number cards with 1, 2, 3 on them. You draw one card *without replacement*, then the second one is drawn. Let  $X$  be the minimal number on both cards, and  $Y$  – the maximal number. Write down the joint PDF for  $X$  and  $Y$  as a table. (Hint: determine your sample space and find possible values of  $X$  and  $Y$ .)

**Problem 3.** (25 pts)

Suppose the PDF of the continuous random variable  $X$  is given by the formula  $f_X(x) = c(x+1)$ ,  $x \in [0, 1]$ , for some unknown parameter  $c$ .

(a) (5 pts) Find the value of the constant  $c$ . (Remember the definition of PDF?)

(b) (5 pts) Find the CDF  $F_X(x)$  for  $X$ . (Hint: consider different cases of  $x$ .)

(c) (10 pts) Compute the probability that  $X \in [-0.5, 0.5]$ . (Use either PDF or CDF)

(d) (5 pts) Find the expected value and variance of  $X$ .

**Problem 4.** (15 pts)

Suppose the joint PDF of the continuous random variables  $X, Y$  is given by the formula  $f_{X,Y}(x, y) = c(x + y)$ ,  $x, y \in [0, 1]$ , for some unknown parameter  $c$ .

(a) (5 pts) Find the value of the constant  $c$ .

(b) (5 pts) Find the marginal pdfs;

(c) (5 pts) Compute the probability that  $X \in [0, 1/2]$ .

**Problem 5.** (5 pts)

Let  $(X, Y)$  be the coordinate of a random point picked from the square with vertices  $(1,0), (0,1), (1,2), (2,1)$ . Find  $P(X \leq 1/2)$ . (Hint: it is geometric probability.)

**Problem 6.** (5 pts)

Let  $X$  be a normal variable with a mean  $\mu = 212$  and a standard deviation  $\sigma = 44.6$ . If you take a random sample of size 80, find  $P(\bar{X} < 220)$ , where  $\bar{X}$  is a sample mean. (Hint: what are the mean and variance of  $\bar{X}$  ?)

**Problem 7.** (10 pts)

38% of people in the country have type O+ blood. If you randomly chose 400 people, use the *normal approximation with continuity correction* to estimate (where  $X$  is the number with type O+):  $P(160 \leq X \leq 200)$ . (Hint: what distribution has  $X$  originally?)

**Problem 8.** (5 pts)

A store gets 15 customers per hour on average. If the number of customers is Poisson, find the probability that the store gets five in the next 12 minutes.

**Problem 9.** (5 pts)

Given a sample  $X_1, \dots, X_{30}$  of size 30 from the continuous distribution with PDF given in Problem 3 approximate the value of  $P(1/4 \leq \bar{X} \leq 3/4)$ . (Hint: you already should have expected value and variance!)