

Name _____

Instructor _____

Show all work clearly.

No.	1, 2	3, 4, 5	6, 7	8, 9	10, 11	12, 13	14, 15	16	Total
	12	13	13	12	12	13	13	12	100

1. (7 pts) 60% of the population received the flu vaccine last winter. The probability of catching the flu was 0.2 for a vaccinated person, and was 0.5 for an unvaccinated person. If a person did not catch the flu, what is the probability he had been vaccinated?

2. (5 pts) Find b if the pdf for the continuous random variable X is

$$f_X(x) = b + x, \quad 0 \leq x \leq b$$

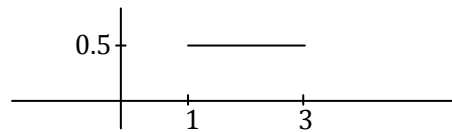
3. (4 pts) Suppose the cdf of a discrete random variable X is

$$F_X(k) = \frac{k^2}{16}, \quad k = 1, 2, 3, 4$$

Find the pdf of X , fill in the following table, with numbers.

k	1	2	3	4
$p_X(k)$				

4. (5 pts) Let X be a random variable with the following pdf. Find the cdf of X .



5. (4 pts) Let X and Y be random variable with the same pdf. Given that $E(X) = E(XY) = 2$, show that X and Y are negatively correlated.

6. (5 pts) Given that X and Y are independent continuous random variables with uniform distribution in $[-1, 1]$, find $P(X + Y \leq 1)$.

7. (8 pts) Suppose the joint pdf of the random variables X and Y are

$$f_{X,Y}(x, y) = \frac{1}{2}xy, \quad 0 < x < y < 2$$

- a) (4 pts) Find the marginal pdf of Y .

- b) (4 pts) Set up an integral, complete with integration limits, to find $P(Y > 2X)$, but do **NOT** evaluate it.

8. (6 pts) The length of a large batch of 1-inch nails produced by a manufacturing process has a normal distribution with mean $\mu = 1$ and $\sigma = 0.04$. If you pick 10 nails, what is the probability that at least 8 of them have length between 0.95 and 1.05?

9. (6 pts) A portable system runs on a battery of which the lifetime is a random variable with mean 10.5 hours and standard deviation 2 hour. Forty batteries are installed in the system so that as soon as the active battery dies, the next battery takes over. Use the Central Limit Theorem (normal approximation) to find the probability that the system runs for at least 400 hours.

10. (8 pts) X is a discrete random variable with the pdf

$$p_X(k; \theta) = \frac{\theta^{2k}}{k!} e^{-\theta^2} \quad k = 0, 1, 2, \dots$$

where θ is an unknown parameter. Find the maximum likelihood estimate (**MLE**) for θ based on a random sample of size n : k_1, \dots, k_n .

11. (4 pts) The Probability and Statistics quiz 1 scores are reported on a scale from 55 to 100. The distribution of quiz scores are approximately Normal with mean $\mu = 80$ and standard deviation $\sigma = 6$. What quiz scores make up the top 15% of all scores?

14. (5 pts) A group wants to know what percent of people can run a mile without stopping. How large a sample will they need if they want a margin of error less than 2% at the 90% confidence level?
15. (8 pts) In a study of life expectancy, a random sample of 9 men lived an average of 68.2 years with a sample standard deviation of 6.2 years, and a random sample of 11 women lived an average of 70.5 years with a sample standard deviation of 7.7 years.
- a) (6 pts) Test at the 0.05 significance level to see if the mean life expectancies are the same for men and women. Assume two populations have the equal variance. You should give your null and alternate hypotheses, then decide if you accept or reject the null hypothesis, by using the **test statistic**.
- b) (2 pts) Construct a 90% confidence interval for the difference between the two means. Do **not** assume that the two populations have the same variance.

16. (12 pts) Suppose the sales of a product per day is a normal variable with mean 400 and standard deviation 100. The company launches an ad campaign. They will look at a sample of 50 days to see if the ad campaign increases sales. They test at the 5% level of significance and use

$$H_0: \mu = 400$$

$$H_1: \mu > 400$$

- a) (5 pts) Find the critical value and use it to get a rejection test.

- b) (5 pts) Find the probability of committing a Type II error if the real mean is 450.

- c) (2 pts) Give the power of the test if the real mean is 450.