

Name _____

- Show work.
- Use the following formula for Question 6

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

1. (8 pts) A box contains 3 number cards, with 1, 2, 3 on them. One card is drawn. Let X be the number on that card. A second card is drawn from the remaining two cards in the box. Let Y be the second card which is drawn. Write down the joint pdf for X and Y as a 2-way table.

2. (12 pts) In an exercise, we found the following joint pdf, where X is the number of heads on the last flip, and Y is the total number of heads in on 3 flips of a fair coin.

$X \backslash Y$	0	1	2	3
0	1/8	1/4	1/8	0
1	0	1/8	1/4	1/8

- a) (4 pts) Write the marginal pdf's $p_X(x)$ and $p_Y(y)$ on the margins of the joint pdf.
b) (8 pts) Calculate the **covariance** of X and Y . Explain why its sign makes sense.

3. (6 pts) Find c , given that the pdf of X is

$$f_X(x) = c(2 - x), \quad 0 \leq x \leq 1$$

4. (8 pts) Find the cdf of X , given that the pdf of X is

$$f_X(x) = \begin{cases} -x & -1 \leq x \leq 0 \\ x & 0 < x \leq 1 \end{cases}$$

5. (8 pts) The cdf of X is

$$F_X(x) = \begin{cases} 0, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

- a) (4 pts) Find $P(1/3 \leq x \leq 1/2)$ from the cdf.
- b) (4 pts) Find the pdf of X .
6. (8 pts) The pdf of X is
- $$f_X(x) = 4xe^{-2x}, \quad x > 0$$
- a) (3 pts) Find $E(X)$. (Use the formula on the cover page.)
- b) (5 pts) Find $Var(X)$. (Use the formula on the cover page.)
7. (8 pts) The joint pdf of X and Y is
- $$f_{X,Y}(x,y) = \frac{1}{3} + xy, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 2$$
- a) (4 pts) Find $f_X(x)$.
- b) (4 pts) Set up a double integral (complete with limits) to find $P(Y \geq 2X)$, but do **NOT** evaluate it.
8. (6 pts) Let (X, Y) be the coordinate of a random point picked from the triangle with vertices $(0,0), (3,0), (0,3)$. Find $P(X \leq 2)$.

(Extra credit 3 pts) Without calculation, give the sign of $Cov(X, Y)$ and explain how you know.

9. (6 pts) The number of major snowstorms has a Poisson distribution with rate 0.4 per month from January through March. Find the probability there are 2 or fewer snow storms during the 3-month period.

10. (6 pts) In a class of 100 students, use **Poisson approximation** to find the probability that at least one person is born on July 4th. Assume that there are 365 days in every year.

11. (6 pts) The volume of a bottle of soft drink is normally distributed with mean 20 oz and standard deviation 1.5 oz. What is the value x such that 95% of the bottles have at least x oz?

12. (9 pts) Suppose the height of people in Country A has a normal distribution with mean 5.4 and standard deviation 0.3, while the height of people in Country B has a normal distribution with mean 5.8 and standard deviation 0.6. What is the probability that a random person in Country A is taller than a random person in Country B?

13. (9 pts) The weekly repair cost for a certain machine is a random variable with mean \$95 and standard deviation \$25. Suppose the budget for repair is \$5200 for a year (52 weeks). Use Central Limit Theorem to estimate the probability that the actual repair cost exceeds the budgeted amount.