MATH 3081 Test 1 Solutions Summer 1 2018

1. (8 pts) Suppose events A and B are **disjoint** (mutually exclusive), and $P(A) = a$ and $P(B) = b$.

   a) (2 pts) Find $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) = a + b$$

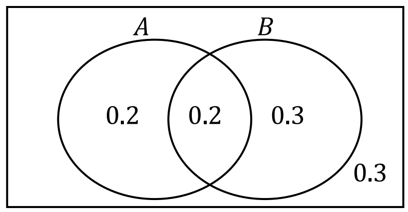
   b) (3 pts) Find $P(A^C \cap B^C)$.

$$P(A^C \cap B^C) = P((A \cup B)^C) = 1 - (a + b) = 1 - a - b$$

   a) (3 pts) Find $P(A^C \cup B^C)$.

$$P(A^C \cup B^C) = P((A \cap B)^C) = 1 - 0 = 1$$

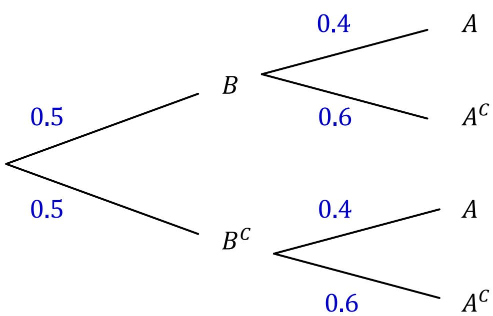
2. (14 pts) The following Venn diagram gives probabilities involving events A and B.



   a) (4 pts) Are A are B disjoint? Justify your answer.
   No, because $P(A \cap B) = 0.2 \neq 0$.

   b) (4 pts) Are A are B independent? Justify your answer.
   Yes, because $P(A|B) = 0.2/0.5 = 0.4 = P(A)$, or $P(A \cap B) = 0.2 = P(A)P(B) = (0.4)(0.5)$.

   c) (6 pts) Complete the following tree diagram by putting the corresponding probabilities on all the branches.



3. (10 pts) Martha brings her children Jerry and Amy, and five of the kids' friends, to movie. To avoid dispute, Martha has everyone including herself draw log to determine the seating (in a row) randomly. Jerry hopes to sit between Eli and Cheryl. What is the probability that it will happen?

   If we treat Eli, Jerry, and Cheryl as a group, there are 6 groups, and 6! ways to arrange the groups. In addition, Eli and Cheryl can switch place.

$$\frac{6!\,2}{8!} = \frac{1}{28} \approx 0.0357$$

4. (10 pts) A candidate for a citizenship test is given a list of 50 questions to prepare. He will be asked 3 questions and he needs to answer at least 2 questions correctly to pass. Suppose he is only prepared for 40 questions. What is the probability he will pass?

$$\frac{\binom{40}{2}\binom{10}{1} + \binom{40}{3}}{\binom{50}{3}} = \frac{9880 + 7800}{19600} = .902$$

5. (12 pts) Suppose 0.5% of a population have criminal ties. The government plans to implement widespread wiretaps. It is known that if a person has criminal ties, the wiretaps will always pick him up. However if the person does not have criminal ties, the wiretaps will wrongly pick him up with an error rate of 1%.

   a) (6 pts) What percentage of the population will be picked up by the wiretaps?

$$(.005)(1) + (.995)(.01) = 0.1495$$

b) (6 pts) Given that a person is picked up by the wiretaps, what is the probability he does **not** have criminal ties?

$$\frac{(.995)(.01)}{(.005)(1)+(.995)(.01)} = 0.6656$$

6. (16 pts) Anna and Beena are shooting the hoop (basketball hoop) from a particular position. Anna's probability of hitting is 25% while Beena's is 50%. It is agreed that Anna will shoot 4 times while Beena will shoot 2 times. Assume that every attempt is independent.

a) (8 pts) What is the probability that Anna has at least 3 hits?

$$\binom{4}{3}(.25^3)(.75)+(.25)^4 = 0.0508$$

b) (8 pts) What is the probability that Anna and Beena have the same number of hits?

Let X be the number of hits for Anna and Y be the number of hits for Benna.

$$P(X=2)P(Y=2)+P(X=1)P(Y=1)+P(X=0)P(Y=0)$$

$$=\binom{4}{2}(.25^2)(.75^2)(.5^2)+\binom{4}{1}(.25)(.75)^3\binom{2}{1}(.5^2)+(.75)^4(.5^2) = .34278$$

7. (10 pts) Three players, A, B, C, play a game with an unbiased three-way spinner. The first player to spins a "1" wins. Find the probability that player A wins.

$$\frac{1}{3}+\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)+\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)+\cdots=\frac{1}{3}\left(\frac{1}{1-\frac{8}{27}}\right)=\frac{1}{3}\left(\frac{27}{19}\right)=\frac{9}{19}$$

8. (10 pts) Let X be number on the top face of a biased die. The cdf of X is given as follows.

k	1	2	3	4	5	6
$F_X(k)$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{4}{9}$	$\frac{2}{3}$	$\frac{7}{9}$	1

a) (6 pts) Find the pdf of X. Write as a table.

k	1	2	3	4	5	6
$p_X(k)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{2}{9}$

b) (4 pts) Roll the biased die twice. Let Y be the largest number. Find $F_Y(3)$.

$$F_Y(3)=P(Y\le 3)=P(X\le 3)P(X\le 3)=\left(F_X(k)\right)^2=\frac{4}{9}\left(\frac{4}{9}\right)=\frac{16}{81}$$

9. (10 pts) Consider a series of coin flips with a biased coin, whose probability of turning up heads is p. Let X be the number of flips until the first head.

a) (3 pts) Find $P(X=1)$.

$$P(X=1)=p$$

b) (3 pts) Find $P(X=2)$.

$$P(X=2)=(1-p)p$$

c) (4 pts) Find a formula for $P(X=k)$, $k=1,2,3,\dots$

$$P(X=k)=(1-p)^{k-1}p, \qquad k=1,2,3,\dots$$