Math 3081 Spring 2017 **Final Exam** NAME: Answers

**Show your work!**

**1.** Let the continuous random variable $X$ have pdf $f_X(x) = c(x^2 + x)$ for $0 \le x \le 2$. Find:

**a)** (3 points) c.

$$1 = c\int_0^2 x^2 + x \, dx = c\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_0^2\right) = \frac{14}{3}c$$

$$\rightarrow \underline{c = 3/14}$$

**b)** (4 points) E(X).

$$= \frac{3}{14}\int_0^2 x\,(x^2 + x)\,dx = \frac{3}{14}\left(\frac{1}{4}x^4 + \frac{1}{3}x^3 \Big|_0^2\right)$$

$$= \underline{10/7}$$

**c)** (4 points) Var(X).

$$E(X^2) = \frac{3}{14}\int_0^2 x^2\,(x^2 + x)\,dx = \frac{3}{14}\left(\frac{1}{5}x^5 + \frac{1}{4}x^4 \Big|_0^2\right)$$

$$= \frac{78}{35} = \underline{2.286}$$

$$Var(X) = E(X^2) - (E(X))^2 = \frac{46}{245} = \underline{.188}$$

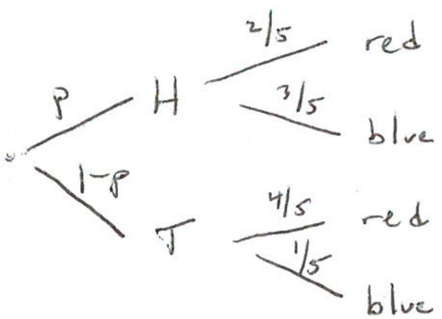
**2.** (3 points) We take a sample and find a 95% confidence interval for the mean of $(24.2, 28.4)$. If we are testing $H_0 : \mu = 24$ against $H_1 : \mu > 24$ at the 5% level of significance, what is our conclusion? Choose from: reject $H_0$, fail to reject $H_0$, and can't tell.

If $H_1 : \mu \neq 24$, we would reject $H_0$ so p-value $< .05$

$\Rightarrow$ for $H_1 : \mu > 24$, p-value $< .025$ $\rightarrow$ reject $H_0$

**3.** Toss an unbalance coin: if it comes up heads choose a chip from urn A; else choose one from urn B.
urn A has 2 red and 3 blue chips;
urn B has 4 red and 1 blue chip.

**a)** (5 points) Find P(getting a blue chip). Let the probability of getting heads be p.




$\rightarrow \frac{3}{5}p + \frac{1}{5}(1-p)$

$= \frac{1}{5} + \frac{2}{5}p$

**b)** (4 points) If P($H$| blue)=.5, find p.

$$= \frac{P(H \cap \text{blue})}{P(\text{blue})} = \frac{\frac{3}{5}p}{\frac{1}{5} + \frac{2}{5}p} = \frac{3p}{1+2p} = .5$$

$\rightarrow 6p = 1 + 2p \rightarrow 4p = 1 \quad p = 1/4$

**4.** (6 points) A roulette wheel is spun 380 times. A roulette wheel has 18 red spaces, 18 black, and 2 other spaces. Estimate the probability that red comes up at least 170 times. (Include a continuity correction in your estimate.)

$P(X \geq 170) \rightarrow P(X \geq 169.5)$

mean $= 380\left(\frac{18}{38}\right) = 180$, $\sigma = \sqrt{380\left(\frac{18}{38}\right)\left(\frac{20}{38}\right)} = 9.73$

$\Rightarrow \approx P\left(Z \geq \frac{169.5 - 180}{9.73} = -1.08\right) = 1 - .1401$

$= \underline{.8599}$

(calc $\rightarrow$ .8597)

**5.** (4 points) The weight of tomatoes at a store is normally distributed with a mean of 8 ounces and a standard deviation of 3.5. If 25 are chosen at random, what is the probability that the mean weight is more than 9 ounces?

$= P(\bar{X} \geq 9) = P\left(Z \geq \frac{9-8}{3.5/\sqrt{25}} = 1.43\right)$

$= 1 - .9236 = \underline{.0764}$

(calc. $\rightarrow$ .0766)

**6.** (4 points) A sample of 24 finds a mean of 94.3. If $\sigma = 12.4$, find a 99% confidence interval for the population mean.

$Z = 2.576 \quad \rightarrow \quad 94.3 \pm 2.576\left(\frac{12.4}{\sqrt{24}}\right)$

$\rightarrow \underline{94.3 \pm 6.52}$

or $(87.78, 100.82)$

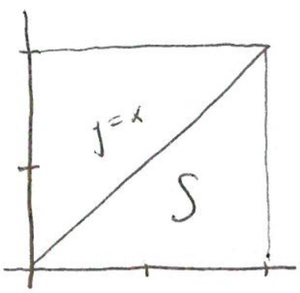
**7.** (4 points) VHL disease occurs in one in 35,000 people. If we randomly look at 100,000 people, use the Poisson distribution to estimate the probability that exactly 3 have VHL. Give your answer to three decimal places.

$$\lambda = \text{mean} = 100000\left(\frac{1}{35000}\right) = 20/7$$

$$\Rightarrow P(X=3) = \frac{e^{-20/7}\left(\frac{20}{7}\right)^3}{3!} = \underline{.223}$$

**8.** Suppose $f_{X,Y}(x,y) = \frac{3}{14}(xy+x)$ for $0 \le x \le 2$ and $0 \le y \le x$.

**a)** (4 points) Find $E(XY)$. Show your work.




$$= \frac{3}{14}\int_0^2\int_0^x xy\,(xy+x)\,dy\,dx$$

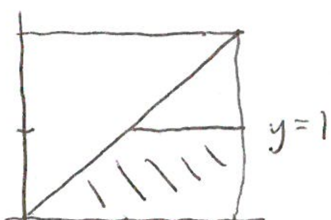
$$= \frac{3}{14}\int_0^2\int_0^x x^2y^2 + x^2y\ \,dy\,dx$$

$$= \frac{3}{14}\int_0^2 \frac{1}{3}x^2y^3 + \frac{1}{2}x^2y^2\Big|_0^x\,dx$$

$$= \frac{3}{14}\int_0^2 \frac{1}{3}x^5 + \frac{1}{2}x^4\,dx = \frac{3}{14}\left(\frac{1}{18}x^6 + \frac{1}{10}x^5\Big|_0^2\right)$$

$$= \underline{\frac{152}{105} = 1.448}$$

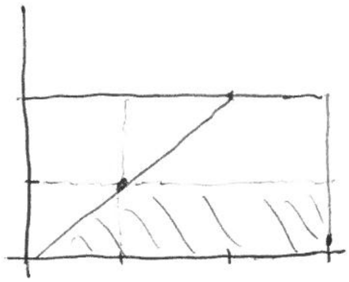
**b)** (5 points) Find $P(Y < 1)$. Set up the integral only.




$$= \underline{\frac{3}{14}\int_0^1\int_y^2 xy + x\ \cancel{dy\,dx}\ dx\,dy}$$

$$\text{or}\quad \frac{3}{14}\left(\int_0^1\int_0^x xy+x\ dy\,dx + \int_1^2\int_0^1 xy+x\ dy\,dx\right)$$

**9.** (4 pts) Suppose that we pick a random point $(x, y)$ from the region: $0 < x < 3$, $0 < y < 2$, and $y < x$ (where all the points are equally likely). Find $P(min(X, Y) < 1)$.



area of $S = \frac{1}{2}(2)(2) + 2 = 4$

shaded area $= \frac{1}{2} + 2 = 5/2$

$\rightarrow P(\min(X, Y) < 1) = \frac{5/2}{4} = \boxed{\frac{5}{8}}$

**10.** (8 points) Suppose the random variables $X_1, X_2, \ldots X_n$ are independent each with the distribution

$f(x; \theta) = \dfrac{\theta x^{\theta - 1}}{3^\theta}$ for $0 \le x \le 3$.

Find the Maximum Likelihood estimate for $\theta$.

$$L(\theta) = \prod_{i=1}^{n} \frac{\theta x_i^{\theta-1}}{3^\theta} = \frac{\theta^n}{3^{n\theta}} \left( \prod_{i=1}^{n} x_i \right)^{\theta - 1}$$

$$\ln(L(\theta)) = n \ln \theta - n\theta \ln 3 + (\theta - 1) \ln\left( \prod_{i=1}^{n} x_i \right)$$

$$\frac{d}{d\theta}\left(\ln(L(\theta))\right) = \frac{n}{\theta} - n \ln 3 + \ln\left( \prod_{i=1}^{n} x_i \right) \overset{\text{set}}{=} 0$$

$$\rightarrow \frac{n}{\theta} = n \ln 3 - \ln\left( \prod_{i=1}^{n} x_i \right)$$

$$\rightarrow \hat{\theta} = \frac{n}{n \ln 3 - \ln\left( \prod_{i=1}^{n} x_i \right)}$$

or $\sum_{i=1}^{n} \ln(x_i)$

**11.** A study looked to see if using integrated pest management, IPM, (as opposed to more conventional pesticide use) would change the condition of children with asthma. 25 children living where IPM was used had a mean FEV (a measure of lung capacity) of 88.9 with a standard deviation of 8.5; 23 children living where there was conventional pesticide use had a mean FEV of 85.9 with a standard deviation of 5.4.
**a)** (5 points) find a 95% confidence interval for the difference between the two means. Assume the variances are NOT the same.

$df = \min(24, 22) = 22 \rightarrow t = 2.074$

$\rightarrow (88.9 - 85.9) \pm 2.074\sqrt{\frac{8.5^2}{25} + \frac{5.4^2}{23}}$

$\rightarrow \underline{3 \pm 4.23}$

calc $\rightarrow (-1.118, 7.118)$

**b)** (2 points) if we test $H_0 : \mu_{IPM} = \mu_{CP}$ against $H_0 : \mu_{IPM} \neq \mu_{CP}$ at the 5% level, then do we accept or reject $H_0$? Use your answer from part (a).

<u>fail to reject $H_o$</u>

**12.** (6 points) A state agency is testing to see if the scales at a chain of supermarkets are accurate. They have a 32 ounce object weighed at a sample of 30 stores and find a mean of 32.1 ounces with a standard deviation of 0.21. Test at the 5% significance level to see if the scales give the correct weight or if they give a larger weight.
You should give your null and alternate hypotheses, then decide if you accept or reject the null hypothesis (find the test statistic and either the p-value or critical value(s)).

$H_o : \mu = 32 \quad , \quad H_1 : \mu > 32$

$t = \frac{32.1 - 32}{.21/\sqrt{30}} = \underline{2.608}$

$df = 29, .05 \rightarrow t = \underline{1.699}$

or T-Test $\rightarrow$ p-valve = .007

$\Bigg\}$ <u>reject $H_o$</u>

**13.** (4 points) A company wants to estimate the percent of people who enter their store end up buying something. If they want a margin of error of at most 2% at the 95% confidence level, then how large a sample should they look at?

$$z = 1.96 \quad \rightarrow \quad n \geq \frac{1}{4}\left(\frac{1.96}{.02}\right)^2 = \underline{2401}$$

**14.** (7 points) A magazine is testing two types of computers to see if they crash at different rates. A sample of 500 of computer A finds that 8% crash when run through a rigorous set of programs. A sample of 500 of computer B finds that 12% crash when run through the same set of programs. Test at the 1% significance level to see if the percents are the same.

You should give your null and alternate hypotheses, then decide if you accept or reject the null hypothesis (find the test statistic and either the p-value or critical value(s)).

$$H_0 : p_A = p_B, \quad H_1 : p_A \neq p_B$$

$$\hat{p}_c = \frac{500(.08) + 500(.12)}{1000} = .1$$

$$z = \frac{.08 - .12}{\sqrt{(.1)(.9)\left(\frac{1}{500} + \frac{1}{500}\right)}} = \underline{-2.11}$$

$$\left.\begin{array}{l} z^*, .005 \rightarrow z = 2.576 \\ \text{2-Prop 2Test} \rightarrow \text{p-value} = .035 \end{array}\right\} \quad \underline{\text{fail to reject } H_0}$$

**15.** A researcher wants to see if the mean number of people in a household in the US is decreasing. They will look at a sample of 500 and assume $\sigma = 1.2$ minutes. If they are testing at the 5% level of significance and use

$H_0 : \mu = 2.59$ and $H_1 : \mu < 2.59$, then:

**(a)** (5 points) find the critical values and use it to get a rejection test.

$z^*, .05 \rightarrow z = 1.645$

$\rightarrow -1.645 = \dfrac{\bar{x} - 2.59}{1.2/\sqrt{500}}$

$\rightarrow \bar{x} = 2.502$

reject $H_0$ if $\bar{x} < 2.502$

**(b)** (5 points) if the power was 90%, what is the real mean?

$\text{Power} = P(\bar{x} < 2.502, \mu) = .9 \rightarrow z = 1.28$

$\rightarrow 1.28 = \dfrac{2.502 - \mu}{1.2/\sqrt{500}} \rightarrow \mu = 2.433$

**16.** (4 points) Suppose $A$, $B$, $C$ are independent events such that $P(B) = .5$, $P(C) = .4$, and $P(A \cup (B \cap C)) = .44$. Find $P(A)$.

$= P(A) + P(B \cap C) - P(A \cap B \cap C)$

$= P(A) + P(B)P(C) - P(A)P(B)P(C)$

$= P(A) + .2 - .2P(A) = .44$

$\rightarrow .8P(A) = .24 \rightarrow P(A) = .3$