Math 3081 Probability and Statistics Summer 1, 2016

Final Exam

Name _____

Instructor _____

There are 10 questions, equally weighted. Show all work. Answers must be supported by calculations or clear reasoning.

For instructor use

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2	
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10	
Total	

1. A test for a disease correctly diagnoses a diseased person as having the disease with probability 0.85. The test incorrectly diagnoses someone without the disease as having the disease with a probability of 0.10. If 1% of the people in a population have the disease, what is the chance that a person from this population who tests positive for the disease actually has the disease?

2. A computer LCD screen contains 1 million (that is, 10^6) pixels, each of which has an independent probability p of being 'dead' due to manufacturing defects. LCD panels are considered accepted if they have at most 3 dead pixels.
   a) Suppose $p = 10^{-6}$. What is the percentage of LCD panels produced that are unacceptable?
   Hint: Use Poisson approximation.

   b) What is the largest p can be if the factory must produce on average at least 50% of panels with no dead pixels?

3. In a particular year, LSAT scores were normally distributed with mean 520 and standard deviation 100.
   a) A selective law school will only admit applicants who scored in the top 9% on the LSAT. What score is required to be admitted?

   b) Five students who took the test are selected randomly. What is the probability that at least one student is in the top 9%?

4. Let X_1 and X_2 be **independent** random variable with the following probability density function.

$$f(x) = \begin{cases} 2 - 2x & 0 < x < 1 \\ 0 \end{cases}$$

   a) Find the probability that X_1 exceeds $1/2$.

   b) Find the probability that exactly one of the two variables exceeds $1/2$.

5. A manufacturing process produces widgets whose length is a random variable, with uniform distribution between 2.0 and 4.0. Let $\bar{X}$ be the average length of a batch of 400 widgets. Use central limit theorem to estimate the probability that $\bar{X}$ exceeds 3. **Note**: you may use the formula for the mean and variance of the uniform distribution derived in class.

6. Suppose there are 4 cards in a box, two with number 0 and two with number 1 written on them. Two cards are randomly drawn without replacement. Let X number on the first card and Y be the number on the second card.
   a) Write the joint pdf as a 2-dimensional table of X and Y, and calculate the marginal pdf's.

   b) Calculate the **covariance** of X and Y. Explain why its sign makes sense.

7. Let Y be a continuos random variable with the pdf

$$f_Y(y;\theta) = \frac{1}{\theta+1} e^{-y/(\theta+1)}, \qquad y > 0$$

where $\theta > -1$ is an unknown parameter. Find the maximum likelihood estimate (**MLE**) for θ based on a random sample of size n: $Y_1, \ldots, Y_n$.

8. A manufacturer of automatic washers offers a model in two colors: white or black. Of the first 1000 washers sold, 567 are white.
   a) Would you conclude that customers in general have a preference for a color? Test with a 99% confidence interval for the unknown proportion (of the population who prefer white).

   b) To reduce by half the margin of error of the estimate for the unknown proportion, what should be the sample size?

9. Two sets of elementary school children were taught to read by using different methods, 15 by each method. At the conclusion of the instructional period, a reading test yielded the results $\bar{y}_1 = 76$, $\bar{y}_2 = 71$, $s_1 = 9$, and $s_2 = 10$.
    a) Find the **test statistic** and use it to test at $\alpha = 0.05$ to see if the two methods yield different results.

    b) Find a 95% confidence interval for the difference of the means.

10. The output voltage for an electric circuit is specified to be 130. A sample of 35 independent readings on the voltage for this circuit gave a sample mean 129.3 and standard deviation of 2.1.
    a) Explain why we may assume that the distribution of sample mean is normal.

    b) Find lowest significance level at which the null hypothesis will be rejected.

    c) Suppose $\alpha = 0.01$ and the true mean is 129. Find the Type II error β.