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1. (a) $\hat{\mu}_X = 8.8$, $S_X = 2.77489$.
   (b) $n-1 = 4$ df. 80% CI: $t_{\alpha/2} = 1.5332$ yielding $(6.89734, 10.70266)$. 95% CI: $t_{\alpha/2} = 2.7764$ yielding $(5.35352, 12.24548)$.
   (c) One-sample t test. $H_0 : \mu = 4$, $H_a : \mu > 4$. Test statistic $t = 3.86795$, p-value is $P(T_4 \geq 3.86795) = 0.00901$, reject null hypothesis.
   (d) $n-1 = 4$ df. 80% CI: $\chi^2_{\alpha/2} = 1.0636$, $\chi^2_{1-\alpha/2} = 7.7794$ yielding $(3.95915, 28.95762)$. 95% CI: $\chi^2_{\alpha/2} = 0.4844$, $\chi^2_{1-\alpha/2} = 11.1433$ yielding $(2.73400, 63.58138)$.
   (e) χ^2 test for variance. $H_0 : \sigma^2 = 100$, $H_a : \sigma^2 < 100$. Test statistic $\chi^2 = 0.308$, p-value is $P(Q_4 \leq 0.308) = 0.01070$, reject null hypothesis.
   (f) Student's equal-variances t test. $H_0 : \mu_X - \mu_Y = 0$, $H_a : \mu_X - \mu_Y \neq 0$. $\hat{\mu}_X = 8.8$, $\hat{\mu}_Y = 7$, $S_X = 2.77489$, $S_Y = 5.59762$, $S_{\text{pool}} = 4.22239$. Test statistic $t = 0.63549$ with $df = 7$, p-value is $p = 2P(T_7 \geq 0.63549) = 0.54532$, fail to reject null hypothesis.
   (g) 7 df, use pooled variance. $\hat{\mu}_X - \hat{\mu}_Y = 1.8$, $S = S_{\text{pool}}\sqrt{\frac{1}{5} + \frac{1}{4}} = 2.83246$. 50% CI: $t_{\alpha/2} = 0.7111$ yielding $(-0.21428, 3.81428)$. 90% CI: $t_{\alpha/2} = 1.89458$ yielding $(-3.56633, 7.16633)$.

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2. (a) One-sample t test (z test is also acceptable since sample size is large). $H_0 : \mu = 20.000$, $H_a : \mu \neq 20.000$. Test statistic $t = -0.62827$ with $n-1 = 99$ df, so p-value is $p = 2P(T_{99} \leq -0.62827) = 0.53127$ [for z-test, p-value is $2P(N_{0,1} \leq -0.62827) = 0.52983$], fail to reject null hypothesis. (Bolt diameter at spec.)
   (b) $n-1 = 99$ df. 90% CI: $t_{\alpha/2} = 1.6604$ yielding $(19.9563\text{mm}, 20.0197\text{mm})$. 99% CI: $t_{\alpha/2} = 2.6364$ yielding $(19.9378\text{mm}, 20.0382\text{mm})$. [Using z instead of t gives CIs $(19.9566\text{mm}, 20.0194\text{mm})$, $(19.9388\text{mm}, 20.0372\text{mm})$.
   (c) χ^2 test for variance. $H_0 : \sigma^2 = 0.0400\text{mm}^2$, $H_a : \sigma^2 < 0.0400\text{mm}^2$. $n-1 = 99$ df. Test statistic $\chi^2 = 90.2905$, p-value is $P(Q_{99} \leq 90.2905) = 0.27748$, fail to reject null hypothesis. (Bolt variance within spec.)
   (d) $n-1 = 99$ df. 90% CI: $\chi^2_{\alpha/2} = 77.0463$, $\chi^2_{1-\alpha/2} = 123.2252$ yielding $(0.1712\text{mm}, 0.2165\text{mm})$. 99% CI: $\chi^2_{\alpha/2} = 66.5101$, $\chi^2_{1-\alpha/2} = 138.9868$ yielding $(0.1612\text{mm}, 0.2330\text{mm})$.

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3. (a) Two-sample t test. $\hat{\mu}_A = 32\text{pp}$, $S_A = 13.6626\text{pp}$, $\hat{\mu}_B = 25\text{pp}$, $S_B = 6.2849\text{pp}$. $H_0 : \mu_A - \mu_B = 0$, $H_a : \mu_A - \mu_B \neq 0$.
   Student: $S_{\text{pool}} = 10.12776$, $df = 7$, test statistic is $t = 1.03034$, p-value is $2P(T_7 \geq 1.03034) = 0.33713$, fail to reject H_0.
   Welch: $S_{\text{unpool}} = 7.29155$, $df = 4.0154$, test statistic is $t = 0.94762$, p-value is $2P(T_{4.0154} \geq 0.94762) = 0.39679$, fail to reject H_0.
   (b) Two-sample t test. $\hat{\mu}_m = 72.2$, $S_m = 14.48012$, $\hat{\mu}_p = 85.16667$, $S_p = 6.70572$. $H_0 : \mu_m - \mu_p = 0$, $H_a : \mu_m - \mu_p < 0$.
   Student: $S_{\text{pool}} = 10.87113$, $df = 9$, test statistic is $t = -1.96978$, p-value is $P(T_9 \leq -1.96978) = 0.04019$, fail to reject H_0.
   Welch: $S_{\text{unpool}} = 7.03096$, $df = 5.4189$, test statistic is $t = -1.84422$, p-value is $P(T_{5.4189} \leq -1.84422) = 0.05998$, fail to reject H_0.
   (c) One-sample t test (matched pairs). $H_0 : \mu_d = 0$, $H_a : \mu_d \neq 0$. $\hat{\mu}_d = 1$, $S_d = 4.95696$, test statistic is $t = 0.57060$, p-value is $P(T_7 \geq 0.57060) = 0.58611$, fail to reject H_0.
   (d) Two-sample t test. $\hat{\mu}_m = 134.\overline{3}$, $S_m = 14.0119$, $\hat{\mu}_b = 181.\overline{6}$, $S_b = 9.6090$. $H_0 : \mu_m - \mu_b = 0$, $H_a : \mu_m - \mu_b < 0$.
   Student: $S_{\text{pool}} = 12.01388$, $df = 4$, test statistic is $t = -4.82536$, p-value is $P(T_4 \leq -4.82536) = 0.004245$, reject H_0.
   Welch: $S_{\text{unpool}} = 9.80929$, $df = 3.5405$, test statistic is $t = -4.82536$, p-value is $P(T_{3.5405} \leq -4.82536) = 0.005734$, reject H_0.

(e) χ^2 test for independence. Comparison table is below. χ^2 statistic is $d = 0.54492$, $df = 3$, p-value is $P(Q_3 \geq 0.54492) = 0.90892$, fail to reject H_0 (seems independent).

Exp (Obs)	USA	UK	China	Russia
Senior	45.47 (46)	25.18 (27)	26.31 (26)	21.04 (19)
Junior	75.53 (75)	41.82 (40)	43.69 (44)	34.96 (37)

(f) Two-sample t test. $H_0 : \mu_m - \mu_f = 0$, $H_a : \mu_m - \mu_f > 0$.
Student: $S_{\text{pool}} = \$22139$, $df = 46$, test statistic is $t = 2.58736$, p-value is $p(T_{46} \geq 2.58736) = 0.00644$, reject H_0.
Welch: $S_{\text{unpool}} = \$6770$, $df = 33.137$, test statistic is $t = 2.52268$, p-value is $p(T_{33.137} \geq 2.52268) = 0.00831$, reject H_0.
Pooled CIs: $\hat{\mu}_{\text{diff}} = \$17078$, $S = S_{\text{pool}}\sqrt{\frac{1}{30} + \frac{1}{18}} = \$6601$, $df = 46$. 80% CI: $t_{\alpha/2} = 1.3002$ yielding $(\$8496, \$25660)$, 95% CI: $t_{\alpha/2} = 2.0129$ yielding $(\$3792, \$30364)$.
Unpooled CIs: $\hat{\mu}_{\text{diff}} = \$17078$, $S = S_{\text{unpool}} = \$6770$, $df = 33.137$. 80% CI: $t_{\alpha/2} = 1.3076$ yielding $(\$8226, \$25930)$, 95% CI: $t_{\alpha/2} = 2.0342$ yielding $(\$3307, \$30849)$.

(g) χ^2 test for independence. Comparison table is below. χ^2 statistic is $d = 16.4934$, $df = 1$, p-value is $P(Q_1 \geq 16.4934) = 0.000049$, reject H_0 (seems non-independent).

Exp (Obs)	Callback	No Callback
White	195.5 (234)	2229.5 (2191)
Black	195.5 (157)	2229.5 (2268)

(h) χ^2 test for goodness of fit. Comparison table is below. χ^2 statistic is $d = 112.52$, $df = 2$, p-value is $P(Q_2 \geq 16.4934) = 3.68 \cdot 10^{-25}$, reject H_0 (definitely not as predicted by model!).

Exp (Obs)	0.001	0.01	0.05
Just Over	104 (94)	98 (76)	183 (87)
Just Under	104 (114)	98 (120)	183 (279)

(i) Two-sample t test. $\hat{\mu}_s = \$200.74$, $S_s = \$22.617$, $\hat{\mu}_c = \$196.69$, $S_c = \$20.353$. $H_0 : \mu_s - \mu_c = 0$, $H_a : \mu_s - \mu_c > 0$.
Student: $S_{\text{pool}} = 21.7394$, $df = 5$, test statistic is $t = 0.24392$, p-value is $P(T_5 \geq 0.24392) = 0.40849$, fail to reject H_0.
Welch: $S_{\text{unpool}} = 16.3082$, $df = 4.7206$, test statistic is $t = 0.24834$, p-value is $P(T_{4.7206} \geq 0.24834) = 0.40715$, fail to reject H_0.
Pooled CIs: $\hat{\mu}_{\text{diff}} = \$4.05$, $S = S_{\text{pool}}\sqrt{\frac{1}{30} + \frac{1}{18}} = \$16.604$, $df = 5$. 50% CI: $t_{\alpha/2} = 0.7267$ yielding $(-\$8.02, \$16.12)$, 80% CI: $t_{\alpha/2} = 1.4759$ yielding $(-\$20.46, \$28.56)$.
Unpooled CIs: $\hat{\mu}_{\text{diff}} = \$4.05$, $S = S_{\text{unpool}} = \$16.308$, $df = 4.7206$. 50% CI: $t_{\alpha/2} = 0.7300$ yielding $(-\$7.85, \$15.95)$, 80% CI: $t_{\alpha/2} = 1.4891$ yielding $(-\$20.23, \$28.33)$.

(j) χ^2 test for goodness of fit. Expected values are all 100. For (i), χ^2 statistic is $d = 0.42$, $df = 9$, p-value is $P(Q_9 \geq 0.42) = 0.9999857$, fail to reject H_0. For (ii), these do seem to close to the model. Testing for "too good" results gives p-value $P(Q_9 \leq 0.42) = 0.000014$, reject H_0 (model is too accurate, suggests forgery).

(k) χ^2 test for goodness of fit. Expected values are 1000 times Benford probability (i.e., just read as 301, 176, 125, etc.). For (i), χ^2 statistic is $d = 13.4150$, $df = 8$, p-value is $P(Q_8 \geq 13.4150) = 0.09835$, fail to reject H_0. For (ii), these results are not suspiciously accurate – it seems finally that the ex-pollster ex-accountant is doing their job properly!