1. (a) (i) $\theta^4(1-\theta)^{14}$ (ii) $\frac{4}{\theta} - \frac{14}{1-\theta}$ (iii) $\hat{\theta} = 4/18$.
   (b) (i) $\theta^{14}e^{-3\theta}/518400$ (ii) $\frac{14}{\theta} - 3$ (iii) $\hat{\theta} = 14/3$.
   (c) (i) $\theta^{-3}e^{-8/\theta}$ (ii) $\frac{-3}{\theta} + \frac{8}{\theta^2}$ (iii) $\hat{\theta} = 8/3$.
   (d) (i) $e^{-(1-\theta)^2-(-2-\theta)^2-(0-\theta)^2}\pi^{-3/2}$ (ii) $2(1-\theta)+2(-2-\theta)+2(0-\theta)$ (iii) $\hat{\theta} = -1/3$.
   (e) (i) $4(\theta-2)(\theta-5)/\theta^4$ (ii) $\frac{1}{\theta-2} + \frac{1}{\theta-5} - \frac{4}{\theta}$ (iii) $\hat{\theta} = 8$ (there is a second root $\hat{\theta} = 5/2$ but it is less than 5).

2. (a) $E(\hat{\lambda}_1) = \frac{1}{2}[E(x_1)+E(x_2)] = \lambda$, unbiased, $\text{var}(\hat{\lambda}_1) = \frac{1}{4}[\text{var}(x_1)+\text{var}(x_2)] = \lambda/2$.
   (b) $E(\hat{\lambda}_2) = \frac{1}{4}[E(x_1)+E(x_2)+E(x_3)+E(x_4)] = \lambda$, unbiased, $\text{var}(\hat{\lambda}_2) = \frac{1}{16}[\text{var}(x_1)+\text{var}(x_2)+\text{var}(x_3)+\text{var}(x_4)] = \lambda/4$.
   (c) $E(\hat{\lambda}_3) = E(x_1)-E(x_2)+2E(x_3) = 2\lambda$, biased, $\text{var}(\hat{\lambda}_3) = \text{var}(x_1)+\text{var}(x_2)+4\text{var}(x_3) = 6\lambda$.
   (d) $E(\hat{\lambda}_4) = \frac{1}{5}[E(x_1)+E(x_2)+2E(x_3)+E(x_4)] = \lambda$, unbiased, $\text{var}(\hat{\lambda}_4) = \frac{1}{25}[\text{var}(x_1)+\text{var}(x_2)+4\text{var}(x_3)+\text{var}(x_4)] = 7\lambda/25$.
   (e) The estimator $\hat{\lambda}_2$ is the most efficient since its variance is the smallest.

3. (a) By expected value properties, $E(\hat{\mu}) = E(ax)+E(by) = aE(x)+bE(y) = (a+3b)\mu$,
   By variance properties, $\text{var}(\hat{\mu}) = \text{var}(ax)+\text{var}(by) = a^2\text{var}(x)+b^2\text{var}(y) = 4a^2+3b^2$.
   (b) Need $a+3b=1$ for unbiased, so $a = 1-3b$. Then $\text{var}(\hat{\mu}) = 4(1-3b)^2+3b^2$ is minimal at $b = 4/13$, so $a = 1/13$.

4. (a) $\mu = 1.6$, $\sigma = 4$, 80%: $(-0.6926, 3.8926)$, 90%: $-1.3425, 4.5425)$, 95%: $(-1.9062, 5.1062)$, 99.5: $(-3.4213, 6.6213)$.
   (b) p-value is $2P(N_{0,4/\sqrt{5}} > 8/5) = 0.3711$, fail to reject at 10%, 3%, 1%.
   (c) $\mu = 1.6$, $S = 3.0496$, 80%: $(-0.4910, 3.6910)$, 90%: $(-1.3075, 4.5075)$, 95%: $(-2.1866, 5.3866)$, 99.5: $(-6.0341, 9.2341)$.

5. (a) H_0: $\mu_c = \mu_p$, H_a: $\mu_c > \mu_p$, $\sigma = \sqrt{\frac{1}{330}+\frac{1}{200}} = 0.0896$, p-value is $P(N_{0,0.0896} > -0.02) = 0.5883$, fail to reject H_0.
   Alternatively, with H_0: $\mu_c = \mu_p$, H_a: $\mu_c < \mu_p$, the p-value is $P(N_{0,0.0896} < -0.02) = 0.4117$, fail to reject H_0.
   (b) H_0: $\mu_t = \mu_p$, H_a: $\mu_t > \mu_p$, $\sigma = \sqrt{\frac{1}{350}+\frac{1}{200}} = 0.0886$, p-value is $P(N_{0,0.0886} > 0.07) = 0.2147$, fail to reject H_0.
   (c) H_0 :$\mu_c = \mu_t$, H_a : $\mu_c \neq \mu_t$, $\sigma = \sqrt{\frac{1}{350}+\frac{1}{330}} = 0.0767$, p-value is $2P(N_{0,0.0767} > 0.09) = 0.2406$, fail to reject H_0.

6. (a) $\mu = 0.38$, $\sigma = 0.0686$, 80%: $(0.2920, 0.4680)$, 90%: $(0.2671, 0.4929)$, 95%: $(0.2455, 0.5145)$, 99%: $(0.2032, 0.5568)$.
   (b) 9 times as many students: 450 in total.
   (c) Need $n = \frac{\hat{p}(1-\hat{p})}{(0.02/1.9600)^2} \approx 2262.7$ (so 2263). If $\hat{p}$ is unknown then worst case is $\hat{p} = 0.5$ with $n = 2401$.
   (d) H_0: $p = 0.3$, H_a: $p \neq 0.3$, p-value is $P(|B_{50,0.3} - 15| \geq |19-15|) \approx 2P(N_{15,3.2404} > 18.5) = 0.2801$, fail to reject H_0.
   (e) H_0: $p = 0.5$, H_a: $p < 0.5$, p-value is $P(B_{50,0.5} < 19) \approx P(N_{25,3.5355} < 19.5) = 0.0599$, reject / fail to reject / fail to reject H_0.
   (f) (d-i) correct / correct / correct, (d-ii) type II / type II / type II,
   (e-i) type I / correct / correct, (e-ii) correct / type II / type II.

7. (a) H_0: $p_c = p_p$, H_a: $p_c > p_p$, $p_{\text{pool}} = 0.3134$, $\sigma_{\text{pool}} = 0.08015$, $\hat{p}_c - \hat{p}_p = 0.02985$, p-value is $P(N_{0,0.08015} > 0.02985) = 0.3548$, fail to reject H_0.

   (b) H_0: $\mu = 0$, H_a: $\mu > 0$, $\sigma/\sqrt{n} = 0.071\text{C}$, p-value is $P(N_{0,0.071\text{C}} > 0.95\text{C}) = 4 \cdot 10^{-41}$, reject H_0 (massively).

   (c) H_0: $\mu_m = \mu_f$, H_a: $\mu_m > \mu_f$, $\mu_{m-f} = 17078$, $\sigma_{\text{pool}} = 6708$, p-value is $P(N_{0,6708} > 17078) = 0.0054$, reject H_0.
   95% CI for male: $(102711, 114471)$, 95% CI for female: $(79753, 103273)$. Note $\sigma_{\text{male,avg}} = 30000/\sqrt{100} = 3000$ and $\sigma_{\text{female,avg}} = 30000/\sqrt{25} = 6000$.

   (d) H_0: $p_w = p_b$, H_a: $p_w > p_b$, $p_{\text{pool}} = 0.0808$, $\sigma_{\text{pool}} = 0.00783$, p-value is $P(N_{0,0.00783} > 0.03176) = 2.5 \cdot 10^{-5}$, reject H_0.
   95% CI for White: $(8.47\%, 10.82\%)$, 95% CI for Black: $(5.49\%, 7.45\%)$. Note $\hat{p}_{\text{White}} = 9.69\%$, $\sigma_{\text{White,prop}} = 0.71\%$, $\hat{p}_{\text{Black}} = 6.47\%$, $\sigma_{\text{Black,prop}} = 0.50\%$.

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8. (a) H_0: $p = 0.156$, H_a: $p \neq 0.156$, $np = 24.8$, $\sqrt{np(1-p)} = 4.575$, p-value is $\approx 2P(N_{24.8,4.575} > 29.5) = 0.3043$, fail to reject H_0.

   (b) H_0: $p = 0.156$, H_a: $p < 0.156$, $np = 24.8$, $\sqrt{np(1-p)} = 4.575$, p-value is $\approx P(N_{24.8,4.575} < 29.5) = 0.8479$, fail to reject H_0.

   (c) H_0: $p = 0.156$, H_a: $p > 0.156$, $np = 24.8$, $\sqrt{np(1-p)} = 4.575$, p-value is $\approx P(N_{24.8,4.575} > 29.5) = 0.1521$, reject H_0 / fail to reject H_0 / fail to reject H_0.

   (d) (i) The study cannot prove anything definitively. It also provides essentially zero evidence toward the hypothesis that hydroxychloroquine is effective in lowering the hospitalization rate, from test (b).
   (ii) The study cannot prove anything definitively. It provides somewhat weak evidence toward the hypothesis that hydroxychloroquine increases the hospitalization rate, from test (c).
   (iii) This is also not entirely accurate, because test (c) does provide weak evidence suggesting that hydroxychloroquine increases the hospitalization rate.

   (e) Yes. The p-values would shift to 0.0752, 0.9624, 0.0376. The power of the test, and hence the strength of the conclusions, increases.

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9. These are all t confidence intervals. Use the t-table with $n-1$ degrees of freedom to get $t_{\alpha/2,n}$, measuring the number of standard deviations in the margin of error.

   (a) $\mu = 200.74$, $S = 22.6166$, 50%: $(192.09, 209.39)$, 80%: $(182.22, 219.26)$, 90%: $(174.13, 227.35)$, 99%: $(134.69, 266.79)$.

   (b) $\mu = 74.2$, $S = 3.4254$, 50%: $(73.44, 74.96)$, 80%: $(72.70, 75.70)$, 90%: $(72.21, 76.19)$, 99%: $(70.68, 77.72)$.

   (c) $\mu = 134.33$, $S = 14.0119$, 50%: $(127.73, 140.94)$, 80%: $(119.08, 149.59)$, 90%: $(110.71, 157.96)$, 99%: $(54.04, 214.62)$.

   (d) $\mu = 109333$, $S = 23459$, 50%: $(98000, 120000)$, 80%: $(84000, 135000)$, 90%: $(70000, 149000)$, 99%: $(-25000, 244000)$.

   (e) $\mu = 201000$, $S = 55323$, 50%: $(189000, 231000)$, 80%: $(165000, 255000)$, 90%: $(145000, 275000)$, 99%: $(48000, 372000)$.

   (f) $\mu = 47000$, $S = 19937$, 50%: $(40000, 54000)$, 80%: $(33000, 61000)$, 90%: $(28000, 66000)$, 99%: $(6000, 88000)$.
