

Answers are given in exact form when the exact form is fairly simple, and otherwise are given to 4 decimal places.

- (a) 0.1      (b) 0.7      (c) 0.3      (d) 0.2      (e) 0.2, 0.5, 0.3 for  $X = 1, 3, 4$       (f) 0.3, 0.4, 0.3 for  $Y = 3, 4, 5$   
 (g) 0.8      (h) 0.7      (i) 2.9 and 4.0      (j) 10.9      (k) 1.09 and 1.0440      (l) 0.6 and 0.7746      (m) No  
 (n) 0.4 and 0.4946
- (a)  $13/24$       (b)  $11/216$  and 0      (c)  $3/8$  and  $5/8$       (d) 0 for  $x < 0$ ,  $(12x^2 - x^3)/216$  for  $0 \leq x \leq 6$ , 1 for  $x > 6$   
 (e)  $7/2$  and 19      (f)  $43/20$  and 1.4663
- (a)  $1/27$       (b)  $2/27$       (c)  $16/27$       (d)  $28/81$       (e)  $16/81$       (f) 0      (g)  $(2x + 6)/27$  for  $0 \leq x \leq 3$   
 (h)  $(2y + 1)/6$  for  $0 \leq y \leq 2$       (i)  $5/3$  and  $11/9$       (j)  $7/2$  and  $16/9$       (k) 2      (l)  $13/18$  and 0.8498  
 (m)  $23/81$  and 0.5329      (n) No      (o)  $-1/27$  and  $-0.0818$
- (a) -1      (b) 7      (c) 24      (d) 9 and 3      (e) 4 and 2      (f) 1      (g)  $1/6$       (h) 15
- (a)  $P(N_{0,1} < 2) = 0.9772$ .      (b)  $P(-1 \leq N_{0,1} \leq 1) = 0.8413 - 0.1587 = 0.6826$ .  
 (c)  $P(N_{0,1} > 1.5) = 1 - 0.9332 = 0.0668$ .      (d)  $E(2X + 5) = 2\mu + 5 = 25$       (e)  $\sigma(2X + 5) = 2\sigma = 4$ .
- (a) Distribution is binomial,  $n = 1800$  and  $p = 2/3$ , so  $P(Y = 1200) = \binom{1800}{1200} \cdot (\frac{2}{3})^{1200} \cdot (\frac{1}{3})^{600}$  and  
 $P(1200 \leq Y \leq 1250) = \binom{1800}{1200} \cdot (\frac{2}{3})^{1200} \cdot (\frac{1}{3})^{600} + \binom{1800}{1202} \cdot (\frac{2}{3})^{1202} \cdot (\frac{1}{3})^{598} + \dots + \binom{1800}{1250} \cdot (\frac{2}{3})^{1250} \cdot (\frac{1}{3})^{550}$ .  
 (b)  $E(Y) = np = 1200$  and  $\sigma(Y) = \sqrt{np(1-p)} = 20$ .  
 (c) Approximate by a normal distribution  $N$  with  $\mu = 1200$  and  $\sigma = 20$ . With continuity correction, get  
 $P(1199.5 \leq N_{1200,20} \leq 1250.5) = P(-0.025 \leq N_{0,1} \leq 2.525) \approx 0.5042$ .
- (a) Distribution is binomial,  $n = 10$  and  $p = 0.4$ , so  $P(\# \geq 2) = 1 - \binom{10}{0}0.6^{10} - \binom{10}{1}0.4^1 0.6^9 \approx 0.9476$ .  
 (b) Distribution is binomial,  $n = 100$  and  $p = 0.8$ , so  $E(\text{pts}) = np = 80$  and  $\sigma(\text{pts}) = \sqrt{np(1-p)} = 4$ .  
 (c) Exp values, variances add for independent variables. Then  $E(2\text{pt}) = 0.4 \cdot 2 = 0.8$ ,  $E(3\text{pt}) = 0.2 \cdot 3 = 0.6$ ,  
 $\text{var}(2\text{pt}) = 2^2 \cdot 0.4 \cdot 0.6 = 0.96$ ,  $\text{var}(3\text{pt}) = 3^2 \cdot 0.2 \cdot 0.8 = 1.44$ . So  $E(\text{sum}) = 1.4$ ,  $\text{var}(\text{sum}) = 2.4$ , so  $\sigma(\text{sum}) = \sqrt{2.4}$ .  
 (d) Distribution will be approximately normal by central limit theorem. Mean is  $1000 \cdot 0.8 + 2000 \cdot 0.8 + 500 \cdot 0.6 = 2700$  points, variance is  $1000 \cdot 0.8 \cdot 0.2 + 2^2 \cdot 2000 \cdot 0.4 \cdot 0.6 + 3^2 \cdot 500 \cdot 0.2 \cdot 0.8 = 2800$  so  $\sigma = \sqrt{2800} \approx 52.92$  points.
- (a) Average A is normal with mean  $\mu_A = 10.05\text{g}$  and  $\sigma_A = 0.10/\sqrt{10} = 0.0316\text{g}$   
 Average B is normal with mean  $\mu_B = 10.05\text{g}$  and  $\sigma_B = 0.10/\sqrt{25} = 0.02\text{g}$ .  
 (b) 0.1586      (c)  $1 - 0.8414^{10} = 0.8223$       (d) 100.5g and 0.3162g      (e) 0.0569      (f) 0.00078  
 (g) Difference is normal with mean  $\mu_{A-B} = \mu_A - \mu_B = 0$  and  $\sigma_{A-B} = \sqrt{\sigma_A^2 + \sigma_B^2} = 0.0374\text{g}$       (h) 0.0907
- (a)  $2/7$       (b) 3.5min and 2.0207min      (c) Approximately normal by central limit theorem with  $\mu = 3.5\text{min}$   
 and  $\sigma = 0.2333\text{min}$       (d) 0.3821      (e) 0.0196 using the normal approximation to the binomial (exact is 0.0120)
- (a) Distribution is Poisson (it is counting rarely-occurring events) with parameter  $\lambda = 3$ ,  $p_X(n) = e^{-\lambda} \lambda^n / n!$   
 (b)  $e^{-3} \approx 0.0498$       (c) 0.1847      (d) This is Poisson with average  $\lambda = 3/7$  so prob. is  $1 - e^{-3/7} \approx 0.3486$   
 (e) Poisson with average  $\lambda = 90/7$  so prob. is  $e^{-90/7} (90/7)^{15} / 15! \approx 0.0865$
- (a) Distribution is exponential (it is a memoryless wait time) with  $\lambda = 1/30$ , so  $p_Y(y) = \lambda e^{-\lambda y}$  for  $y \geq 0$   
 (b)  $e^{-1} \approx 0.3689$       (c)  $1 - e^{-1/3} \approx 0.2835$
- (a)  $0.98^{130} + \binom{130}{1} 0.02^1 0.98^{129} = 0.2643$   
 (b) Parameter is the average number who like the course, which is  $130 \cdot 2\% = 2.6$ . The resulting probability estimate is  $P(X = 0) + P(X = 1) = e^{-2.6} + 2.6e^{-2.6} \approx 0.2674$ .  
 (c) Parameters are the mean and standard deviation, which are  $np = 2.6$  and  $\sqrt{np(1-p)} = 1.5962$ . The continuity-corrected probability estimate is  $P(-0.5 \leq N \leq 1.5) \approx 0.2193$ .  
 (d) The Poisson is better since the normal approximation to the binomial is not especially good for  $np$  small. On the other hand, this is precisely the situation where the Poisson distribution is a good approximation.