Lecture: Monday-Wednesday-Thursday, 4:35pm–5:40pm, 330 Dodge.

Instructor: Evan Dummit (he/him/his), 571 Lake Hall, edummit@northeastern.edu.

Instructor Office Hours: Monday noon–1:00pm and Monday/Thursday 3:00pm-4:00pm, or by appointment.

Course Webpage: https://dummit.cos.northeastern.edu/teaching_sp26_4571.html.

Course Textbook: The instructor will provide lecture notes for the course in lieu of an official textbook. For additional reference, Friedberg/Insel/Spence's "Linear Algebra" covers topics at a similar level.

Prerequisites: Math 1365 (Introduction to Mathematical Reasoning) or an equivalent familiarity with proof, and either Math 2331 (Linear Algebra) or Math 2341 (Differential Equations and Linear Algebra) or an equivalent familiarity with matrices.

Course Philosophy: Math 4571 is a second course in linear algebra that covers the material from a theoretical perspective. Linear algebra is a fundamental stepping stone to almost every area of advanced mathematics (especially abstract algebra, functional analysis, algebraic topology, and applied and computational mathematics) and the sciences (especially machine learning, theoretical physics, complex systems analysis, and applied engineering). The primary theme of Math 4571 is to study vector spaces and linear transformations from a formal standpoint. Many applications of linear algebra treat the theoretical aspects like a "black box": the goal of Math 4571 is to gain a deep understanding of the fundamental objects (vector spaces, linear transformations, inner products, quadratic forms) in linear algebra, in addition to their uses in a wide variety of applications

Proof is a central theme in this course, and the homework assignments and exams will emphasize proofs and problem-solving. The homework is an integral component of the course and, as such, it is expected that all students will work on it every week: the purpose is to require complex problem solving and combining multiple ideas together in novel ways as a way of solidifying the foundation created during the lectures.

Grades and Exams: Your course grade consists of **25% homework and 75% exams.**

The homework score is the average of the written assignment scores, with the lowest score dropped.
The exam score is the maximum of [20% each midterm and 35% final] and [30% best midterm, 45% final].

An overall raw score of 92% will be **at least** an A, 90% will be **at least** an A-, 88% will be **at least** a B+, 82% will be **at least** a B, 80% will be **at least** a B-, 78% will be **at least** a C+, 72% will be **at least** a C, and 70% will be **at least** a C-.

If you feel that an assignment or exam has been misgraded, please talk to the instructor directly. Requests for regrading will not be considered more than two days past the date the assignment or exam was returned.

Exams: There will be two 1-hour midterm exams, along with a 2-hour final exam. If you miss an examination for any reason, you will receive a 0; make-up exams will not be given other than in exceptional circumstances.

The midterms are in class and scheduled for Wednesday, February 18th and Wednesday, April 1st.

The final exam will occur during the final exam week, date and time TBA.

Homework: Written assignments will be assigned weekly and due via Gradescope by 11:59pm on the due date. **It is highly recommended to start work on the assignments early**: many problems will require substantial thought and effort to solve, even if the solution is ultimately fairly short. There is a **30-hour late period** during which assignments may still be submitted, possibly with a late penalty assessment. The lowest assignment grade is dropped, to provide you a cushion if an emergency arises and you cannot complete an assignment.

Problem sessions will be held weekly on Tuesdays and Fridays. The problem sessions provide you a place to work collaboratively on the homework assignments with help from the TAs. **It is highly recommended to start the assignments early**: many problems will require substantial thought and effort to solve, even if the solution is ultimately fairly short. **Do not fall into the trap of only starting the assignment the evening before it is due!**

Written assignments should be organized carefully, neatly, and in complete sentences, with concise well-reasoned logical arguments. Cite any external resources used, and clearly label all problems. **Failure to adhere to any of these guidelines may result in point deductions, at the grader's discretion.**

Collaboration Policy: Mathematics is fundamentally a collaborative endeavor, and discussing the course material with others is an excellent way to solidify your own understanding. However, it is critical not to outsource your learning! You cannot expect to retain knowledge if you do not solve your homework problems yourself (e.g., if you relied on other people or on technology to explain how to do the problems). **On written assignments, you may work together with other people, but you must write up your work independently**.

Course Schedule: The course and lecture notes are organized into six chapters, as follows:

Weeks 1-2: Chapter 0 $\sim$ Preliminaries: Vectors, matrices, determinants, fields, polynomials, induction

Weeks 3-4: Chapter 1 $\sim$ Vector Spaces: Definition, subspaces, span, linear independence, bases and dimension

Weeks 5-6: Chapter 2 $\sim$ Linear Transformations: Kernel and image, the nullity-rank theorem, algebraic properties, matrices associated to linear transformations, isomorphisms and inverses, change of basis and similarity

Weeks 7-9: Chapter 3 $\sim$ Inner Products: Inner products, norms, Cauchy-Schwarz, orthogonality, Gram-Schmidt, orthogonal projections and complements, linear functionals, adjoints, least-squares approximation, Fourier series

Weeks 10-12: Chapter 4 $\sim$ Eigenvalues, Diagonalization, and the Jordan Canonical Form: Eigenvalues, eigenvectors, the characteristic polynomial, the Cayley-Hamilton theorem, diagonalization, generalized eigenvectors, the Jordan canonical form and its applications, Markov chains, Hermitian operators and the spectral theorem

Weeks 12-14: Chapter 5 $\sim$ Bilinear and Quadratic Forms: Bilinear forms, diagonalization, quadratic forms, the second derivatives test, Sylvester's law of inertia, singular value decomposition, pseudoinverses.

Resources and AI Policy: If you use **any** external resources (e.g., wikipedia, stackexchange, other books beyond the course text or notes, other people, etc.) you must say what results you are citing and where they are from. **If you happen to find a solution to an assigned problem online or elsewhere, it is plagiarism to present it as your own work without attribution of its source.**

Use of generative AI / large language models (e.g., ChatGPT, Copilot) or similar technology, is expressly prohibited in this course. This includes asking for hints or solutions to assigned problems, checking solutions, searching or summarizing course-related content, and outlining drafts of submitted work.

Although the course lectures and notes provide a basic introduction to the course material, you will not actually understand the ideas until you have used them to solve new problems yourself. **Using generative AI to assist with solving the homework problems thus short-circuits your learning**, because you will not have worked through the problems as fully as you should have. Additionally, while generative AI technology produces confident and authoritative answers, it in fact often makes mistakes both small and large.

The instructor reserves the right to require any student to explain any submitted work in person; failure or inability to do so satisfactorily will be considered evidence that the work is not your own. **If plagiarism or AI use is detected on an assignment, you will receive a warning and an automatic zero on the assignment. Another violation after the warning will result in an automatic F in the course.**

Attendance Policy: It is expected that you will attend every class. This course moves very fast, and it is quite possible to fall behind even if you only miss one day. If you miss class for any reason, it is highly advisable to consult the course lecture notes to catch up, and you may also wish to obtain notes from another student. It is your responsibility to be aware of all information announced in class, including modifications to the course syllabus or schedule, even if you are absent.

If you will be absent from a class activity due to a religious observance or practice, or for participation in a university-sanctioned event (e.g., university athletics), it is your responsibility to inform the instructor during the first week of class and provide appropriate documentation if required. Your instructor will work with you on alternative and reasonable arrangements for any time missed.

Statement on Academic Integrity: A commitment to the principles of academic integrity is essential to the mission of Northeastern University. Academic dishonesty violates the most fundamental values of an intellectual community and undermines the achievements of the entire University. Violations of academic integrity include (but are not limited to) cheating on assignments or exams, fabrication or misrepresentation of data or other work, plagiarism, unauthorized collaboration, and facilitation of others' dishonesty. Possible sanctions include (but are not limited to) warnings, grade penalties, course failure, suspension, and expulsion.

Statement on Accommodations: Any student with a disability is encouraged to meet with the instructor during the first week of classes to discuss accommodations. The student must bring a current Memorandum of Accommodations from the Office of Student Disability Services.

Statement on Inclusivity: Faculty are encouraged to address students by their preferred name and gender pronoun. If you would like to be addressed using a specific name or pronoun, please let your instructor know.

Statement on Evaluations: Students are requested to complete the TRACE evaluations at the end of the course.

Miscellaneous Disclaimer: The instructor reserves the right to change course policies, including the evaluation scheme of the course. Notice will be given in the event of any substantial changes.