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1. Determine/calculate/find the following:
   (a) The number of residue classes in $\mathbb{Z}[i]$ mod $7+2i$ and $\mathbb{F}_5[x]$ mod x^4+2.
   (b) A fundamental region and list of residue class representatives for $\mathbb{Z}[i]$ modulo $2-i$.
   (c) A prime factorization of $5+5i$ in $\mathbb{Z}[i]$.
   (d) A prime factorization of $11+12i$ in $\mathbb{Z}[i]$.
   (e) A prime factorization of 85 in $\mathbb{Z}[i]$.
   (f) A prime factorization of 999 in $\mathbb{Z}[i]$.
   (g) Which of 104, 224, 420, and 666 are the sum of two squares.
   (h) Two ways of writing $450 = 2 \cdot 3^2 \cdot 5^2$ as the sum of two squares.
   (i) A way of writing $3626 = 2 \cdot 7^2 \cdot 37$ as the sum of two squares.
   (j) Two Pythagorean right triangles with a side length 29.
   (k) Whether 13 and 26 are quadratic residues modulo the prime 2027.
   (l) Whether 28 and 15 are quadratic residues modulo the prime 71.
   (m) Whether 2 is a quadratic residue modulo the primes 67 and 71.
   (n) The values of the Legendre symbols $\left(\frac{103}{307}\right)$ and $\left(\frac{141}{307}\right)$.
   (o) The values of the Jacobi symbols $\left(\frac{47}{245}\right)$ and $\left(\frac{177}{245}\right)$.

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2. Give brief responses justifying the following statements:
   (a) Given that 11291867 is prime, we can quickly determine whether the congruence $x^2 \equiv 3 \pmod{11291867}$ has a solution, even without a computer.
   (b) There is a faster way to solve the congruence $x^2 \equiv 3 \pmod{11291867}$ than simply checking each possible residue class modulo 11291867 to see if it is a solution.
   (c) Because $\left(\frac{31}{6601}\right) = -1$ but $31^{(6601-1)/2} \equiv +1 \pmod{6601}$, that means 6601 must be composite.
   (d) Studying rings such as $\mathbb{Z}[i]$ and $F[x]$ can help answer number theory questions about integers.

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3. Solve the following:
   (a) Prove that there are no elements of norm 2 or -2 in $\mathbb{Z}[\sqrt{26}]$. [Hint: Consider $a^2 - 26b^2 = \pm 2$ mod 13.]
   (b) Prove that $2+\sqrt{26}$ is irreducible in $\mathbb{Z}[\sqrt{26}]$.
   (c) Show that $2+\sqrt{26}$ is not prime in $\mathbb{Z}[\sqrt{26}]$.
   (d) Verify Euler's theorem for the residue class of $1+i$ modulo $4+i$ in $\mathbb{Z}[i]$.
   (e) Prove that there exists a solution to $x^2 \equiv 11 \pmod{97}$. Note 97 is prime.
   (f) Prove that there exists a solution to $x^2 + 6x \equiv 14 \pmod{101}$. Note 101 is prime.
   (g) If $p > 3$ is a prime, prove that 3 is a quadratic residue modulo p if and only if $p \equiv 1, 11 \pmod{12}$.
   (h) If $p > 3$ is a prime, prove that -3 is a quadratic residue modulo p if and only if $p \equiv 1 \pmod{3}$.
   (i) Characterize the primes dividing an integer of the form $n^2 + 4n - 1$, for n an integer.
   (j) Characterize the primes dividing an integer of the form $n^2 + 6n + 11$, for n an integer.
