Lecture: Monday-Wednesday-Thursday, 4:35pm–5:40pm, location TBA.

Instructor: Evan Dummit (he/him/his), 571 Lake Hall, edummit@northeastern.edu.

Instructor Office Hours: Mon 12:05pm-1:05pm (shared) + Thu 3:00pm-4:00pm, Lake 571.

Problem Sessions: Mondays and Fridays, times and locations TBA.

Course Webpage: https://dummit.cos.northeastern.edu/teaching_sp25_4571.html.

Course Textbook: The instructor will write lecture notes for the course (in lieu of an official textbook) as the semester progresses. For students seeking additional reference, "Linear Algebra" (4th or 5th edition) by Friedberg/Insel/Spence and "Linear Algebra Done Right" by Axler are both good resources.

Prerequisites: Math 1365 (Introduction to Mathematical Reasoning) or an equivalent familiarity with proof, and either Math 2331 (Linear Algebra) or Math 2341 (Differential Equations and Linear Algebra) or an equivalent familiarity with matrices.

Course Philosophy: Math 4571 is a second course in linear algebra that covers the material from a theoretical perspective. Linear algebra is a fundamental stepping stone to almost every area of advanced mathematics (especially abstract algebra, functional analysis, algebraic topology, differential geometry, and applied and computational mathematics) and the sciences (especially machine learning and artificial intelligence, theoretical physics, complex systems analysis, and applied engineering). The primary theme of Math 4571 is to study vector spaces and linear transformations from a formal standpoint. Many applications of linear algebra can be treated entirely from a computational perspective, but this approach often leads to treating computations like a "black box": the goal of Math 4571 is to gain a deep understanding of how these computations work, in addition to understanding their applications.

At the end of the course, you will have a solid grasp of the fundamentals of vector spaces, linear transformations, inner products, and quadratic forms, and how they can be used in a wide variety of applications. Success in this course thereby demands facility with the basic concepts, with the underlying theory, and with its applications. Proof is a central theme in this course, and the homework assignments and exams will emphasize proofs and problem-solving. The homework is an integral component of the course and, as such, it is expected that all students will work on it every week: the purpose is to require complex problem solving and combining multiple ideas together in novel ways as a way of solidifying the foundation created during the lectures.

Grades and Exams: Your course grade is determined by your scores on the weekly homework assignments and a take-home midterm and final exam.

There are 12 homework assignments each worth 30 points, with the lowest score dropped, for a total of 330. The midterm exam is worth 90 points, and the final exam is worth up to 120 points.

A total score of at least 420 will guarantee an A, 410 will be **at least** an A-, 400 will be **at least** a B+, 380 will be **at least** a B, 360 will be **at least** a B-, 350 will be **at least** a C+, and 330 will be **at least** a C.

Students with a cumulative total of at least 380 points on the homeworks and midterm are eligible to take the very-short-final exam.

Homework: Written assignments will be assigned weekly and due via Gradescope on **Mondays by 11:59pm** before spring break and **Fridays by 11:59pm** after spring break. There is a **48-hour late period** during which assignments may still be submitted, possibly with a late penalty assessment (at the grader's sole and total discretion) that will represent at most 15% of the assignment maximum score.

Problem sessions will be held weekly on Mondays and Fridays. The problem sessions provide you a place to work collaboratively on the homework assignments with help from the TAs. **It is highly recommended to start the assignments early**: many problems will require substantial thought and effort to solve, even if the solution is ultimately fairly short. **Do not fall into the trap of only starting the assignment the evening before it is due!**

Written assignments should be organized carefully, neatly, and in complete sentences, with concise well-reasoned logical arguments. Cite any external resources used, and clearly label all problems. If you collaborated with any other students, write the names of all collaborators on the top of your assignment. If your assignment is more than one page long, use a staple or paperclip to affix the pages together. **Failure to adhere to any of these guidelines may result in point deductions, at the grader's discretion.**

Course Schedule: The course and lecture notes are organized into six chapters, as follows:

Weeks 1-2: Chapter 0 $\sim$ Preliminaries: Vectors, matrices, determinants, fields, polynomials, induction

Weeks 3-4: Chapter 1 $\sim$ Vector Spaces: Definition, subspaces, span, linear independence, bases and dimension

Weeks 5-6: Chapter 2 $\sim$ Linear Transformations: Kernel and image, the nullity-rank theorem, algebraic properties, matrices associated to linear transformations, isomorphisms and inverses, change of basis and similarity

Weeks 7-9: Chapter 3 $\sim$ Inner Products: Inner products, norms, Cauchy-Schwarz, orthogonality, Gram-Schmidt, orthogonal projections and complements, linear functionals, adjoints, least-squares approximation, Fourier series

Weeks 10-12: Chapter 4 $\sim$ Eigenvalues, Diagonalization, and the Jordan Canonical Form: Eigenvalues, eigenvectors, the characteristic polynomial, the Cayley-Hamilton theorem, diagonalization, generalized eigenvectors, the Jordan canonical form and its applications, Markov chains, Hermitian operators and the spectral theorem

Weeks 12-14: Chapter 5 $\sim$ Bilinear and Quadratic Forms: Bilinear forms, diagonalization, quadratic forms, the second derivatives test, Sylvester's law of inertia, singular value decomposition, pseudoinverses.

Collaboration Policy: Mathematics is fundamentally a collaborative endeavor, and discussing the course material with others is an excellent way to solidify your own understanding. However, it is critical not to outsource your learning! You cannot expect to retain knowledge if you do not solve your homework problems yourself, whether because you relied on other people to explain to you how to do the problems, or because you relied too heavily on technological assistance.

On written assignments, you may work together with other people, **but you must write up your work independently**. Please note, however, that **collaboration is not allowed on the take-home exams**.

External Resources Policy: If you use **any** external resources (e.g., wikipedia, stackexchange, other books beyond the course text or notes, other people, etc.) you must say **what results you are citing and where they are from**. If you happen to find a solution to an assigned problem online or elsewhere, it is plagiarism to present it as your own work without attribution of its source. **In particular, solutions to some homework problems from previous years may be available online: copying from these solutions is prohibited and will be treated as an academic honesty violation.**

Use of generative AI / large language models (e.g., ChatGPT, Copilot) or similar technology, is **expressly prohibited in this course. Submitting answers generated by such models constitutes plagiarism and will receive an automatic zero on the assignment**. Additionally, students are highly discouraged from attempting to check their solutions using generative AI, as the responses are not typically accurate enough to learn effectively from: instead, ask questions on the course Piazza, or speak to the TAs or instructor.

Attendance Policy: It is expected that you will attend every class. This course moves very fast, and it is quite possible to fall behind even if you only miss one day. If you miss class for any reason, it is highly advisable to consult the course lecture notes to catch up, and you may also wish to obtain notes from another student. It is your responsibility to be aware of all information announced in class, including modifications to the course syllabus or schedule, even if you are absent.

If you will be absent from a class activity due to a religious observance or practice, or for participation in a university-sanctioned event (e.g., university athletics), it is your responsibility to inform the instructor during the first week of class and provide appropriate documentation if required. Your instructor will work with you on alternative and reasonable arrangements for any time missed.

Statement on Academic Integrity: A commitment to the principles of academic integrity is essential to the mission of Northeastern University. Academic dishonesty violates the most fundamental values of an intellectual community and undermines the achievements of the entire University. Violations of academic integrity include (but are not limited to) cheating on assignments or exams, fabrication or misrepresentation of data or other work, plagiarism, unauthorized collaboration, and facilitation of others' dishonesty. Possible sanctions include (but are not limited to) warnings, grade penalties, course failure, suspension, and expulsion.

Statement on Accommodations: Any student with a disability is encouraged to meet with the instructor during the first week of classes to discuss accommodations. The student must bring a current Memorandum of Accommodations from the Office of Student Disability Services.

Statement on Inclusivity: Faculty are encouraged to address students by their preferred name and gender pronoun. If you would like to be addressed using a specific name or pronoun, please let your instructor know.

Statement on Evaluations: Students are requested to complete the TRACE evaluations at the end of the course.

Miscellaneous Disclaimer: The instructor reserves the right to change course policies, including the evaluation scheme of the course. Notice will be given in the event of any substantial changes.