Lecture: Monday-Wednesday-Thursday, 10:30am–11:35am, location TBA.

Instructor: Evan Dummit (he/him/his), Lake Hall 571, edummit@northeastern.edu.

Instructor Office Hours: Mon 12:05pm-1:05pm (shared) + Mon 3:00pm-4:00pm, Lake 571.

Problem Sessions: Tuesdays, time and location TBA.

Course Webpage: https://dummit.cos.northeastern.edu/teaching_sp25_4527.html.

Course Textbook: The instructor will write lecture notes for the course (in lieu of an official textbook) as the semester progresses.

Prerequisites: Math 3527 (Number Theory 1) or Math 3150 (Group Theory) or instructor's permission. Some basic abstract algebra (fundamentals of groups and rings) will be incorporated into the course as it progresses, as needed.

Course Philosophy: The goal of this course is to give an overview of some additional topics in number theory, continuing roughly where Math 3527 leaves off. The exact set of topics will depend on student and instructor interest, but the course topics will be drawn from the following list: Farey sequences, continued fractions and rational approximation, Diophantine equations, Pell's equation, transcendence, elliptic curves and the group law, elliptic curve factorization and cryptography, rational points on elliptic curves, quadratic integer rings, factorization in quadratic integer rings, class groups, Minkowski's theorems and the geometry of numbers, binary quadratic forms, the p-adic numbers and their properties, the Riemann zeta function, Dirichlet series, arithmetic functions, L-functions, the analytic class number formula, the prime number theorem, modular forms, modularity, and a brief overview of Fermat's last theorem.

Since this is an advanced undergraduate-level class, much of the learning, and many of the secondary topics, are developed through the homework assignments. The lectures will primarily focus on discussing the results and giving applications. The homework is an integral component of the course and, as such, it is expected that all students will work on it every week: the purpose is to require complex problem solving and combining multiple ideas together in novel ways as a way of solidifying the foundation created during the lectures.

Grades: Your course grade is determined by your scores on the weekly homework assignments and a take-home final exam.

There are 12 homework assignments each worth 30 points, the lowest score of which is dropped, meaning that 330 points are available on the homework assignments. The final exam is worth a maximum of 120 points.

A total score of at least 340 will guarantee an A, 330 will be **at least** an A-, 320 will be **at least** a B+, 300 will be **at least** a B, 280 will be **at least** a B-, 270 will be **at least** a C+, and 250 will be **at least** a C.

Students with a cumulative total of at least 300 points on the homeworks (without dropping any scores) are eligible to take the very-short-final exam.

Homework: Written assignments will be assigned weekly and due via Gradescope on **Tuesdays by 11:59pm**. There is a **48-hour late period** during which assignments may still be submitted, possibly with a late penalty assessment (at the grader's sole and total discretion) that will represent at most 15% of the assignment maximum score.

Problem sessions will be held weekly on Tuesdays. The problem sessions provide you a place to work collaboratively on the homework assignments with help from the TAs. **It is highly recommended to start the assignments early**: many problems will require substantial thought and effort to solve, even if the solution is ultimately fairly short. **Do not fall into the trap of only starting the assignment the evening before it is due!**

Written assignments should be organized carefully, neatly, and in complete sentences, with concise well-reasoned logical arguments. Cite any external resources used, and clearly label all problems. If you collaborated with any other students, write the names of all collaborators on the top of your assignment. If your assignment is more than one page long, use a staple or paperclip to affix the pages together. **Failure to adhere to any of these guidelines may result in point deductions, at the grader's discretion.**

Course Schedule: The course and lecture notes are organized into five chapters, four of which will be covered.

Weeks 1-3: Chapter 6 $\sim$ Rational Approximation and Diophantine Equations: the Farey sequences, finite and infinite continued fractions, rational approximation, irrationality and transcendence, Pell's equation, Pythagorean triples, miscellaneous Diophantine equations.

(Skipped) Chapter 7 $\sim$ Elliptic Curves: elliptic curves, the group law, elliptic curves modulo p, elliptic curve factorization, elliptic curve cryptography, rational points on elliptic curves.

Weeks 4-8: Chapter 8 $\sim$ Quadratic Integer Rings: rings and ideals, Euclidean domains, PIDs and UFDs, quadratic fields and integer rings, ideal factorizations, applications of unique factorization.

Weeks 9-11: Chapter 9 $\sim$ The Geometry of Numbers: Minkowski's theorems, Lagrange's four-squares theorem, the ideal class group, binary quadratic forms, reduction and composition of quadratic forms.

Weeks 12-14: Chapter 10 $\sim$ Analytic Number Theory: the Riemann zeta function, Dirichlet series, Dirichlet's theorem on primes in arithmetic progressions, L-series, the analytic class number formula.

Collaboration Policy: Mathematics is fundamentally a collaborative endeavor, and discussing the course material with others is an excellent way to solidify your own understanding. However, it is critical not to outsource your learning! You cannot expect to retain knowledge if you do not solve your homework problems yourself, whether because you relied on other people to explain to you how to do the problems, or because you relied too heavily on technological assistance.

On written assignments, you may work together with other people, **but you must write up your work independently**. Please note, however, that **collaboration is not allowed on the take-home exams**.

External Resources Policy: If you use **any** external resources (e.g., wikipedia, stackexchange, other books beyond the course text or notes, other people, etc.) you must say **what results you are citing and where they are from**. If you happen to find a solution to an assigned problem online or elsewhere, it is plagiarism to present it as your own work without attribution of its source. **In particular, solutions to some homework problems from previous years may be available online: copying from these solutions is prohibited and will be treated as an academic honesty violation.**

Use of generative AI / large language models (e.g., ChatGPT, Copilot) or similar technology, is **expressly prohibited in this course. Submitting answers generated by such models constitutes plagiarism and will receive an automatic zero on the assignment**. Additionally, students are highly discouraged from attempting to check their solutions using generative AI, as the responses are not typically accurate enough to learn effectively from: instead, ask questions on the course Piazza, or speak to the TAs or instructor.

Attendance Policy: It is expected that you will attend every class. This course moves very fast, and it is quite possible to fall behind even if you only miss one day. If you miss class for any reason, it is highly advisable to consult the course lecture notes and to watch the recording of the lecture you missed. It is your responsibility to be aware of all information announced in class, including modifications to the course syllabus or schedule, even if you are absent.

If you will be absent from a class activity due to a religious observance or practice, or for participation in a university-sanctioned event (e.g., university athletics), it is your responsibility to inform the instructor during the first week of class and provide appropriate documentation if required.

Statement on Academic Integrity: A commitment to the principles of academic integrity is essential to the mission of Northeastern University. Academic dishonesty violates the most fundamental values of an intellectual community and undermines the achievements of the entire University. Violations of academic integrity include (but are not limited to) cheating on assignments or exams, fabrication or misrepresentation of data or other work, plagiarism, unauthorized collaboration, and facilitation of others' dishonesty. Possible sanctions include (but are not limited to) warnings, grade penalties, course failure, suspension, and expulsion.

Statement on Accommodations: Any student with a disability is encouraged to meet with or otherwise contact the instructor during the first week of classes to discuss accommodations. The student must bring a current Memorandum of Accommodations from the Office of Student Disability Services.

Statement on Inclusivity: Faculty are encouraged to address students by their preferred name and gender pronoun. If you would like to be addressed using a specific name or pronoun, please let your instructor know.

Statement on Evaluations: Students are requested to complete the TRACE evaluations at the end of the course.

Miscellaneous Disclaimer: The instructor reserves the right to change course policies, including the evaluation scheme of the course. Notice will be given in the event of any substantial changes.