Lecture: Monday-Wednesday-Thursday, 4:35am–5:40am, Snell Library 033.

Instructor: Evan Dummit, Lake Hall 571, edummit@northeastern.edu.

Office Hours: Thursday noon-1:00pm + 3:00pm-4:00pm, or by appointment, online via Zoom.

Course Webpage: https://web.northeastern.edu/dummit/teaching_sp24_4527.html.

Course Textbook: The instructor will write lecture notes for the course (in lieu of an official textbook) as the semester progresses.

Prerequisites: Math 3527 (Number Theory 1) or Math 3150 (Group Theory) or instructor's permission. Some basic abstract algebra (fundamentals of groups and rings) will be incorporated into the course as it progresses, as needed.

Course Philosophy: The goal of this course is to give an overview of some additional topics in number theory, continuing roughly where Math 3527 leaves off. The exact set of topics will depend on student and instructor interest, but the course topics will be drawn from the following list: Farey sequences, continued fractions and rational approximation, Diophantine equations, Pell's equation, transcendence, elliptic curves and the group law, elliptic curve factorization and cryptography, rational points on elliptic curves, quadratic integer rings, factorization in quadratic integer rings, class groups, Minkowski's theorems and the geometry of numbers, binary quadratic forms, the p-adic numbers and their properties, the Riemann zeta function, Dirichlet series, arithmetic functions, L-functions, the analytic class number formula, the prime number theorem, modular forms, modularity, and a brief overview of Fermat's last theorem.

Since this is an advanced undergraduate-level class, much of the learning, and many of the secondary topics, are developed through the homework assignments. The lectures will primarily focus on discussing the results and giving applications. The homework is an integral component of the course and, as such, it is expected that all students will work on it every week: the purpose is to require complex problem solving and combining multiple ideas together in novel ways as a way of solidifying the foundation created during the lectures.

Grades: Your course grade is determined by your scores on the weekly homework assignments and a take-home final exam.

There are 12 homework assignments each worth 30 points, the lowest score of which is dropped, meaning that 330 points are available on the homework assignments. The final exam is worth a maximum of 120 points.

A total score of at least 320 will guarantee an A, 310 will be **at least** an A-, 300 will be **at least** a B+, 280 will be **at least** a B, 270 will be **at least** a B-, 260 will be **at least** a C+, and 250 will be **at least** a C.

Students with a cumulative total of at least 300 points on the homeworks (with the lowest grade dropped) are eligible to take the very-short-final exam.

Homework Assignments: Written assignments will be assigned weekly and due via Gradescope, typically on Fridays. There will be a weekly problem session on Fridays supervised by the course TA, providing a time and place to work on the homework assignments in small groups and get assistance from the TA.

It is highly recommended to start work on the assignments early: many problems will require substantial thought and effort to solve, even if the solution is ultimately fairly short. Do not fall into the trap of only starting the assignment the evening before it is due!

The lowest assignment grade is dropped, to provide you a cushion if an emergency arises and you cannot complete an assignment. Assignments may be submitted late, but late submissions may be penalized at the grader's discretion.

Written assignments should be organized carefully, neatly, and in complete sentences, with concise well-reasoned logical arguments. Cite any external resources used, and clearly label all problems. All electronic submissions are expected to be easily readable and all pages on which any portion of your response to a given problem appears must be marked when submitting your assignment. **Failure to adhere to any of these guidelines may result in point deductions, at the grader's discretion.**

If you feel that an assignment has been misgraded, you may submit a regrade request on Gradescope. Requests for regrading should be made promptly, within two weeks that the graded assignment was returned to you; regrade requests made after two weeks may be declined at the discretion of the grader.

Course Schedule: The course and lecture notes are organized into five chapters, four of which will be covered.

Weeks 1-3: Chapter 6 $\sim$ Rational Approximation and Diophantine Equations: the Farey sequences, finite and infinite continued fractions, rational approximation, irrationality and transcendence, Pell's equation, Pythagorean triples, miscellaneous Diophantine equations.

(Skipped) Chapter 7 $\sim$ Elliptic Curves: elliptic curves, the group law, elliptic curves modulo p, elliptic curve factorization, elliptic curve cryptography, rational points on elliptic curves.

Weeks 4-7: Chapter 8 $\sim$ Quadratic Integer Rings: quadratic fields, quadratic integer rings, factorization in quadratic integer rings, applications of unique factorization, binary integral quadratic forms, ideal class groups.

Weeks 8-10: Chapter 9 $\sim$ The Geometry of Numbers: Minkowski's theorems, the geometry of numbers, Lagrange's four-squares theorem, binary quadratic forms, reduction and composition of quadratic forms.

Weeks 11-13: Chapter 10 $\sim$ Analytic Number Theory: arithmetic functions, growth rates, the Riemann zeta function, the prime number theorem, modular forms, modularity, survey of Fermat's last theorem.

Collaboration Policy: Mathematics is fundamentally a collaborative endeavor, and discussing the course material with others is an excellent way to solidify your own understanding. However, it is critical not to outsource your learning! You cannot expect to retain knowledge if you do not solve your homework problems yourself, whether because you relied on other people to explain to you how to do the problems, or because you relied too heavily on technological assistance.

On written assignments, you may work together with other people, but you must write up your work independently. If you use **any** external resources (e.g., wikipedia, stackexchange, other books beyond the course text or notes, other people, etc.) you must say **what results you are citing and where they are from**. If you happen to find a solution to an assigned problem online or elsewhere, it is academically dishonest to copy the solution and present it as your own work. **In particular, copying solutions from previous years' assignments is expressly forbidden, and will result in an automatic zero on an assignment.**

Attendance Policy: It is expected that you will attend every class. This course moves very fast, and it is quite possible to fall behind even if you only miss one day. If you miss class for any reason, it is highly advisable to consult the course lecture notes and to watch the recording of the lecture you missed. It is your responsibility to be aware of all information announced in class, including modifications to the course syllabus or schedule, even if you are absent.

If you will be absent from a class activity due to a religious observance or practice, or for participation in a university-sanctioned event (e.g., university athletics), it is your responsibility to inform the instructor during the first week of class and provide appropriate documentation if required.

Statement on Academic Integrity: A commitment to the principles of academic integrity is essential to the mission of Northeastern University. Academic dishonesty violates the most fundamental values of an intellectual community and undermines the achievements of the entire University. Violations of academic integrity include (but are not limited to) cheating on assignments or exams, fabrication or misrepresentation of data or other work, plagiarism, unauthorized collaboration, and facilitation of others' dishonesty. Possible sanctions include (but are not limited to) warnings, grade penalties, course failure, suspension, and expulsion.

Statement on Accommodations: Any student with a disability is encouraged to meet with or otherwise contact the instructor during the first week of classes to discuss accommodations. The student must bring a current Memorandum of Accommodations from the Office of Student Disability Services.

Statement on Classroom Behavior: Disruptive classroom behavior will not be tolerated. In general, any behavior that impedes the ability of your fellow students to learn will be viewed as disruptive.

Statement on Inclusivity: Faculty are encouraged to address students by their preferred name and gender pronoun. If you would like to be addressed using a specific name or pronoun, please let your instructor know.

Statement on Evaluations: Students are requested to complete the TRACE evaluations at the end of the course.

Miscellaneous Disclaimer: The instructor reserves the right to change course policies, including the evaluation scheme of the course. Notice will be given in the event of any substantial changes.