
In all problems, assume each surface is oriented with "upward" or "outward" orientation. Circulation and flux along/across a curve always refer to counterclockwise circulation and outward normal flux.

1. Compute the following:
   (a) $\int_C x^2\,ds$, where C is the line segment from $(0,1)$ to $(3,2)$.
   (b) $\int_C x\,dx + y\,dy$, where C is the circle $x^2+y^2=9$.
   (c) The integral of $\sqrt{x^2+y^2}$ on the portion of the helix $\mathbf{r}(t) = \langle \cos 3t, \sin 3t, 4t\rangle$ for $0 \le t \le \pi$.
   (d) The average value of y on the upper half of the circle $x^2+y^2=4$.
   (e) The work done by $\mathbf{F}(x,y,z) = \langle yz,\, xz,\, x^3\rangle$ N on a particle that travels from $(0,0,0)$ to $(1,1,1)$ along the curve parametrized by $\mathbf{r}(t) = \langle t^2,\, t^4,\, t^3\rangle$ m.
   (f) The (outward normal) flux of $\mathbf{F}(x,y) = \langle y+x,\, y-x\rangle$ across, and the circulation of $\mathbf{F}$ along, the path C along the upper half of $x^2+y^2=1$ from $(1,0)$ to $(-1,0)$.
   (g) The (outward normal) flux of $\mathbf{F}(x,y) = \langle xy,\, y^2\rangle$ across, and the circulation of $\mathbf{F}$ along, the curve $\mathbf{r}(t) = \langle t^2, t^3\rangle$ for $0 \le t \le 1$.

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2. Find an equation for the tangent plane to each surface at the given point:
   (a) The surface parametrized by $\mathbf{r}(s,t) = \langle s+t, s^2+t^2, s^3+t^3\rangle$ where $s=1$ and $t=2$.
   (b) The surface parametrized by $\mathbf{r}(s,t) = \langle s^2, 2st, 3t^3\rangle$ at the point $(x,y,z) = (1,2,-3)$.

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3. Parametrize each surface, and then set up (do not evaluate) an iterated double integral for each of the following:
   (a) $\iint_S (x^2+y^2)\,d\sigma$ where S is the portion of the plane $z = 2x+2y$ above the rectangle with $1 \le x \le 2$, $2 \le y \le 4$.
   (b) $\iint_S (zx^2+zy^2)\,d\sigma$ where S is the portion of the cylinder $x^2+y^2=4$ between $z=0$ and $z=4$.
   (c) $\iint_S \sqrt{x^2+y^2}\,d\sigma$ where S is the portion of the cone $z = 4\sqrt{x^2+y^2}$ below $z=8$.
   (d) $\iint_S z^2\,d\sigma$ where S is the unit sphere.

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4. Set up (do not evaluate) an iterated double integral giving the area of each surface:
   (a) The portion of the plane $z = 3x+4y+11$ above the region with $0 \le x \le 1$ and $0 \le y \le 2$.
   (b) The portion of the surface parametrized by $\mathbf{r}(s,t) = \langle s^2, st, t^2\rangle$ with $0 \le s \le 1$ and $0 \le t \le 2$.
   (c) The portion of the cone $z = 3\sqrt{x^2+y^2}$ that lies between the cylinders $x^2+y^2=4$ and $x^2+y^2=9$.
   (d) The portion of the sphere $x^2+y^2+z^2=9$ that lies below the cone $z = \sqrt{3}\cdot\sqrt{x^2+y^2}$.

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5. For each vector field $\mathbf{F}$ and surface S, compute the outward normal flux $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma$ of $\mathbf{F}$ across S:
   (a) $\mathbf{F} = \langle x^2,\, 2x^2,\, 3x^2\rangle$, where S is the portion of the plane $x+y+z=4$ with $0 \le x \le 1$ and $0 \le y \le 2$.
   (b) $\mathbf{F} = \langle xz,\, yz,\, x^2+y^2\rangle$, where S is the portion of the cone $z = \sqrt{x^2+y^2}$ inside the cylinder $x^2+y^2=6$.
   (c) $\mathbf{F} = \langle -xz,\, -yz,\, z^4\rangle$, where S is the portion of the cylinder $x^2+y^2=4$ between $z=0$ and $z=3$.
   (d) $\mathbf{F} = \langle y,\, -x,\, z\rangle$, where S is the portion of the surface $z = x^2+y^2$ below $z=2$.
   (e) $\mathbf{F} = \langle x, y, z\rangle$, where S is the upper half of the unit sphere.
   (f) $\mathbf{F} = \langle y^3, x^3, 1\rangle$, where S is the portion of the paraboloid $z = 1-x^2-y^2$ above the xy-plane.

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6. For each vector field, (i) find the divergence and curl of $\mathbf{F}$, and (ii) determine whether $\mathbf{F}$ is conservative and (if so) find a potential function U:

   (a) $\mathbf{F} = \langle x^3 + xy,\, y^3 + xy,\, 0 \rangle$.

   (b) $\mathbf{F} = \langle yz + 2x,\, xz + 2z,\, xy + 2y \rangle$.

   (c) $\mathbf{F} = \langle x^2yz,\, x^2z^2,\, 2x^2yz \rangle$.

   (d) $\mathbf{F} = \langle 2xyz + e^z,\, x^2z + 3,\, xe^z + x^2y + 2 \rangle$.

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7. Evaluate $\int_C yz\,dx + xz\,dy + xy\,dz$, where C is the curve $\mathbf{r}(t) = \langle te^t, \arctan(t), \ln(1+t) \rangle$ for $0 \le t \le 1$.

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8. Let C be the curve that runs once counterclockwise around the boundary of the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$. Find $\oint_C (x^2 + y)\,dx + (2xy^2 - xy)\,dy$.

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9. For the given C and $\mathbf{F}$, find the counterclockwise circulation $\oint_C \mathbf{F}\cdot\mathbf{T}\,ds$ and the outward normal flux $\oint_C \mathbf{F}\cdot\mathbf{N}\,ds$.

   (a) $\mathbf{F} = \langle xy^2,\, y^3 \rangle$, C is the boundary of the rectangle with vertices $(0,0)$, $(2,0)$, $(2,3)$, $(0,3)$.

   (b) $\mathbf{F} = \langle 6xy,\, 0 \rangle$, C is the boundary of the triangle with vertices $(0,0)$, $(1,0)$, and $(0,2)$.

   (c) $\mathbf{F} = \langle 2x + 3y,\, 4x + 5y \rangle$, C is the unit circle.

   (d) $\mathbf{F} = \langle x^3 - y^3,\, x^3 + y^3 \rangle$, C is the boundary of the quarter-disc $x^2 + y^2 \le 16$ in the first quadrant.

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10. Let $C = C_1 \cup C_2 \cup C_3$, where C_1 is the line segment from $(-2,0)$ to $(0,0)$, C_2 is the line segment from $(0,0)$ to $(\sqrt{2}, \sqrt{2})$, and C_3 is the shorter arc of the circle $x^2 + y^2 = 4$ from $(\sqrt{2}, \sqrt{2})$ to $(-2,0)$.

    (a) Find the outward flux of $\mathbf{F} = \langle 2x^3y^2 + y^3,\, x^2 - 2x^2y^3 \rangle$ around C.

    (b) Find the counterclockwise circulation of $\mathbf{F} = \langle x^2 - y^3,\, x^3 + y^4 \rangle$ around C.

    (c) Find the work done by $\mathbf{F} = \langle -2y, 2x \rangle$ on a particle that travels once around C.

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11. A particle moves through the force field $\mathbf{F}(x,y) = \left\langle 9y - e^{x^2} + 11,\, 4x + \sin\sqrt{y} \right\rangle$ N where x and y are measured in meters.

    (a) Is the vector field $\mathbf{F}$ conservative? If so find a potential function and if not explain why not.

    (b) Calculate the work done by $\mathbf{F}$ if the particle starts at $(0,0)$, moves along a straight line to $(0,4)$, then moves counterclockwise along the circle $x^2 + y^2 = 16$ to $(-4,0)$, then finally moves in a straight line back to $(0,0)$.

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12. Let $\mathbf{F}(x,y) = \langle 1 - y,\, \cos(y^2) + 2x \rangle$ and let C be the curve starting at $(0,0)$, traveling along a straight line to $(0,3)$, then along a straight line to $(4,3)$, and then along a straight line to $(4,0)$. Find the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$.
