1. (a) System is $1 = 2\lambda x$, $3 = 2\lambda y$, $x^2 + y^2 = 40$. Thus $x = 1/(2\lambda)$, $y = 3/(2\lambda)$, so $10/(4\lambda^2) = 40$ so $\lambda = \pm 1/4$, yielding $(x, y) = (2, 6), (-2, -6)$. Min of -18 at $(-2, -6)$, max of 22 at $(2, 6)$.

   (b) System is $y^2 = 2\lambda x$, $2xy = 2\lambda y$, $x^2 + y^2 = 12$. If $y = 0$ then get points $(\pm\sqrt{12}, 0)$. Otherwise, second equation gives $x = \lambda$, and then first equation gives $y^2 = 2\lambda^2$, so third equation is $3\lambda^2 = 12$ so $\lambda = \pm 2$ and $(x, y) = (\pm 2, \pm\sqrt{8})$. Min of -16 at $(-2, \pm\sqrt{8})$, max of 16 at $(2, \pm\sqrt{8})$.

   (c) System is $y = 3\lambda$, $x = \lambda$, $3x + y = 60$. Thus $6\lambda = 60$ so $\lambda = 10$ yielding $(x, y) = (10, 30)$. Minimum does not exist (f goes to $-\infty$ for large x or large y), maximum is 300 at $(10, 30)$.

   (d) System is $2 = 2\lambda x$, $4 = 2\lambda y$, $5 = 2\lambda z$, $x^2 + y^2 + z^2 = 1$. Thus $x = 2/(2\lambda)$, $y = 4/(2\lambda)$, $z = 5/(2\lambda)$, so $45/(4\lambda^2) = 1$ so $\lambda = \pm\sqrt{45/4}$, yielding $(x, y, z) = \pm\frac{1}{\sqrt{45}}(2, 4, 5)$. Min of $-\sqrt{45}$ at $-\frac{1}{\sqrt{45}}(2, 4, 5)$, max of $\sqrt{45}$ at $\frac{1}{\sqrt{45}}(2, 4, 5)$.

   (e) System is $yz = 2\lambda x$, $xz = 8\lambda y$, $xy = 32\lambda z$, $x^2 + 4y^2 + 16z^2 = 48$. If one variable is zero then so must be a second, yields points $(x, y, z) = (\pm\sqrt{48}, 0, 0)$, $(0, \pm\sqrt{12}, 0)$, $(0, 0, \pm\sqrt{3})$. If none are zero, divide first two equations to get $y/x = x/(4y)$ so $x/y = \pm 2$. Similarly, $x/z = \pm 4$. Yields points $(\pm 4, \pm 2, \pm 1)$ with all sign choices. Min of -8 and max of 8.

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2. We maximize $A = lw$ subject to $2l + w = 60$m. Using Lagrange gives $w = 2\lambda$, $l = \lambda$, $2l + w = 60$m and so $4\lambda = 60$ and $\lambda = 15$. Thus $l = 15$m, $w = 30$m.

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3. (a) $I = \int_0^2 \frac{3}{2} y^4 \, dy = \frac{48}{5}$.   (b) $I = \int_0^1 (x^3 - x^4) \, dx = \frac{1}{20}$.   (c) $I = \int_{\pi/3}^{\pi/2} \frac{7}{3} \sin\theta \, dx = \frac{7}{6}$.

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4. (a) Integrals are $\int_0^1 \int_0^3 x^2 y \, dy \, dx$ and $\int_0^3 \int_0^1 x^2 y \, dx \, dy$.

   (b) Integrals are $\int_0^4 \int_{x^2}^{8\sqrt{x}} (x + y) \, dy \, dx$ and $\int_0^{16} \int_{y^2/64}^{\sqrt{y}} (x + y) \, dx \, dy$.

   (c) Integrals are $\int_0^1 \int_0^x x^3 \, dy \, dx + \int_1^2 \int_0^{2-x} x^3 \, dy \, dx$ and $\int_0^1 \int_y^{2-y} x^3 \, dx \, dy$.

   (d) Integrals are $\int_{-\sqrt{6}}^{\sqrt{6}} \int_{x^2-1}^5 1 \, dy \, dx$ and $\int_{-1}^5 \int_{-\sqrt{y+1}}^{\sqrt{y+1}} 1 \, dx \, dy$.

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5. (a) $\int_0^9 \int_{\sqrt{y}}^3 xy \, dx \, dy$.   (b) $\int_1^2 \int_{\sqrt{x}}^x y^4 \, dy \, dx + \int_2^4 \int_{\sqrt{x}}^2 y^4 \, dy \, dx$.

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6. (a) Region is interior of a quarter-circle. Integral is $\int_\pi^{3\pi/2} \int_0^1 (r\cos\theta) \, r \, dr \, d\theta = \int_\pi^{3\pi/2} \frac{1}{3} \cos\theta \, d\theta = -\frac{1}{3}$.

   (b) Region is interior of a quarter-circle. Integral is $\int_{\pi/4}^{3\pi/4} \int_0^4 (r) \, r \, dr \, d\theta = \int_{\pi/4}^{3\pi/4} \frac{64}{3} \, d\theta = \frac{32\pi}{3}$.

   (c) Region is interior of a circle. Integral is $\int_0^{2\pi} \int_0^2 (4 - r^2) \, r \, dr \, d\theta = \int_0^{2\pi} 12 \, d\theta = 24\pi$.

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7. Region is interior of a quarter-circle. Polar integral is $\int_0^{\pi/2} \int_0^1 \frac{1}{r} \cdot r \, dr \, d\theta = \int_0^{\pi/2} \int_0^1 1 \, dr \, d\theta = \int_0^{\pi/2} 1 \, d\theta = \pi/2$.

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8. Region is interior of a triangle. Reversed integral is $\int_0^4 \int_0^{2y} \frac{e^y}{y} \, dx \, dy = \int_0^4 2e^y \, dy = 2(e^4 - 1)$.

9. (a) $I = \int_0^2 \int_x^{2x} 3(y^2 - x^2)\, dy\, dx = \int_0^2 4x^3\, dx = 16.$ (b) $I = \int_0^\pi \int_0^\pi 4\sin\varphi\, d\varphi\, d\theta = \int_0^\pi 8\, d\theta = 8\pi.$

10. (a) Cylindrical: $\int_0^{2\pi} \int_0^1 \int_0^{r\cos\theta + r\sin\theta} r \cdot r\, dz\, dr\, d\theta = \int_0^{2\pi} \int_0^1 r^3(\sin\theta + \cos\theta)\, dr\, d\theta = \int_0^{2\pi} \frac{1}{4}(\sin\theta + \cos\theta)\, d\theta = 0.$

   (b) Spherical: $\int_0^\pi \int_0^{\pi/2} \int_0^2 \rho \cdot \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta = \int_0^\pi \int_0^{\pi/2} 4\sin\varphi\, d\varphi\, d\theta = \int_0^{\pi/2} 4\, d\theta = 2\pi.$

   (c) Cylindrical: $\int_{-\pi/2}^{\pi/2} \int_0^3 \int_{-1}^{r^2} \frac{1}{r} \cdot r\, dz\, dr\, d\theta = \int_{-\pi/2}^{\pi/2} \int_0^3 (r^2 + 1)\, dr\, d\theta = \int_{-\pi/2}^{\pi/2} 12\, d\theta = 12\pi.$

   (d) Spherical: $\int_0^{2\pi} \int_0^{\pi/4} \int_0^2 \frac{\rho^2 \cos^2\varphi}{\rho} \cdot \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta = \int_0^{2\pi} \int_0^{\pi/4} 4\cos^2\varphi \sin\varphi\, d\rho\, d\varphi\, d\theta = 4\pi(4 - \sqrt{2})/3.$

11. (a) Rectangular: $\int_0^1 \int_0^2 \int_{x^2+y^2}^7 (x^2 + y^2)\, dz\, dy\, dx.$

   (b) Rectangular: $\int_1^2 \int_{-3}^3 \int_{y^2}^9 xyz\, dz\, dy\, dx.$

   (c) Cylindrical: $\int_0^{2\pi} \int_0^{3/2} \int_{2r}^3 (r^2 + z^2)\, r\, dz\, dr\, d\theta.$

   (d) Cylindrical: $\int_{\pi/2}^{3\pi/2} \int_0^2 \int_0^{4 - r\sin\theta} zr \cdot r\, dz\, dr\, d\theta.$

   (e) Rectangular: $\int_1^2 \int_y^{2y} \int_{2x}^{3x} 1\, dz\, dx\, dy.$

   (f) Spherical: $\int_0^{2\pi} \int_0^{5\pi/6} \int_0^2 \rho \cdot \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta.$

   (g) Cylindrical: $\int_0^{2\pi} \int_1^{\sqrt{5}} \int_0^{5 - r^2} 1 \cdot r\, dz\, dr\, d\theta.$

   (h) Spherical: $\frac{1}{4\pi/3} \int_0^{\pi/2} \int_0^{\pi/2} \int_0^2 \rho^2 \cdot \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta.$

   (i) Rectangular: $\int_0^4 \int_0^{\sqrt{4-x}} \int_0^{4-x-y^2} x\, dz\, dy\, dx.$

12. (a) Mass is $M = \int_0^1 \int_0^2 \int_0^3 z\, dz\, dy\, dx = 9$g. Moments are $M_x = \int_0^1 \int_0^2 \int_0^3 xz\, dz\, dy\, dx = \frac{9}{2}$g, $M_y = \int_0^1 \int_0^2 \int_0^3 yz\, dz\, dy\, dx = 9$g, $M_z = \int_0^1 \int_0^2 \int_0^3 z \cdot z\, dz\, dy\, dx = 18$g. Center of mass is $\frac{1}{M}(M_x, M_y, M_z) = (1/2, 1, 2)$cm.

   (b) Mass is $M = \int_0^{2\pi} \int_0^2 \int_0^r 2r \cdot r\, dz\, dr\, d\theta = \frac{128\pi}{5}$g. Moments are $M_x = \int_0^{2\pi} \int_0^2 \int_0^r r\cos\theta \cdot 2r \cdot r\, dz\, dr\, d\theta = 0$, $M_y = \int_0^{2\pi} \int_0^2 \int_0^r r\sin\theta \cdot 2r \cdot r\, dz\, dr\, d\theta = 0$, $M_z = \int_0^{2\pi} \int_0^2 \int_0^r z \cdot 2r \cdot r\, dz\, dr\, d\theta = \frac{64\pi}{3}$. Center of mass is $\frac{1}{M}(M_x, M_y, M_z) = (0, 0, 5/6)$cm.

   (c) Mass is $M = \int_0^{2\pi} \int_0^\pi \int_{\sqrt{2}}^{\sqrt{3}} 3\rho^3 \cdot \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta = 38\pi$kg. Center of mass is $(0, 0, 0)$m by symmetry. Alternatively, could evaluate moments as integrals: $M_x = \int_0^{2\pi} \int_0^\pi \int_{\sqrt{2}}^{\sqrt{3}} \rho\cos\theta \sin\varphi \cdot 3\rho^3 \cdot \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta = 0$, $M_y = \int_0^{2\pi} \int_0^\pi \int_{\sqrt{2}}^{\sqrt{3}} \rho\sin\theta \sin\varphi \cdot 3\rho^3 \cdot \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta = 0$, $M_z = \int_0^{2\pi} \int_0^\pi \int_{\sqrt{2}}^{\sqrt{3}} \rho\cos\varphi \cdot 3\rho^3 \cdot \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta = 0$,

   (d) Mass is $M = \int_0^{2\pi} \int_{\pi/6}^{\pi/4} \int_0^3 1 \cdot \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta = 9(\sqrt{3} - \sqrt{2})\pi$mg. The x and y-coordinates of the center of mass are 0 by symmetry. The z-moment is $M_z = \int_0^{2\pi} \int_{\pi/6}^{\pi/4} \int_0^3 \rho\cos\varphi \cdot \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta = \frac{81\pi}{16}$. Center of mass is $\frac{1}{M}(M_x, M_y, M_z) = (0, 0, \frac{9\sqrt{3}+9\sqrt{2}}{16})$mm.