1. (a) Use the divergence theorem: $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \text{div}(\mathbf{F})\,dV$. The solid is $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$ and $\text{div}(\mathbf{F}) = y^2z^2$. Thus by the divergence theorem, the flux is $\iiint_D \text{div}(\mathbf{F})\,dV = \int_0^1\int_0^1\int_0^1 y^2z^2\,dz\,dy\,dx = \boxed{1/9}$.

   (b) Use the divergence theorem: $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \text{div}(\mathbf{F})\,dV$. The solid is $0 \le r \le 2$, $0 \le \theta \le 2\pi$, $1 \le z \le 3$ in cylindrical and also $\text{div}(\mathbf{F}) = 3x^2z + 3y^2z = 3r^2z$. Thus by the divergence theorem, the flux is $\iiint_D \text{div}(\mathbf{F})\,dV = \int_0^{2\pi}\int_0^2\int_1^3 3r^2z\cdot r\,dz\,dr\,d\theta = \boxed{96\pi}$.

   (c) Use the divergence theorem: $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \text{div}(\mathbf{F})\,dV$. The solid is $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi$, $0 \le \rho \le 1$ in spherical and also $\text{div}(\mathbf{F}) = y^2 + z^2 + x^2 = \rho^2$. Thus by the divergence theorem, the flux is $\iiint_D \text{div}(\mathbf{F})\,dV = \int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho^2\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \boxed{4\pi/5}$.

   (d) Use Stokes's theorem: $\iint_S(\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma = \oint_C P\,dx + Q\,dy + R\,dz$ where C is the boundary of the hemisphere. We can parametrize the boundary by $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 0\rangle$ for $0 \le t \le 2\pi$. Then $dx = -3\sin t\,dt$, $dy = 3\cos t\,dt$, $dz = 0$ and $P = 6\sin t$, $Q = 6\cos t$, $R = 3\cos t\cdot e^{3\sin t}$. Thus by Stokes, the integral is $\int_0^{2\pi}(-6\sin t)\cdot(-3\sin t)\,dt + (6\cos t)\cdot 3\cos t\,dt + 3\cos t\cdot e^{3\sin t}\cdot 0\,dt = \int_0^{2\pi} 18\,dt = \boxed{36\pi}$.

   (e) Use the divergence theorem: $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \text{div}(\mathbf{F})\,dV$. The solid is $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi/4$, $0 \le \rho \le 1$ in spherical and also $\text{div}(\mathbf{F}) = 12$. Thus by the divergence theorem, the flux is $\iiint_D \text{div}(\mathbf{F})\,dV = \int_0^{2\pi}\int_0^{\pi/4}\int_0^1 12\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \boxed{4\pi(2-\sqrt{2})}$.

   (f) Use Stokes's theorem: $\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S(\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$ where S is the portion of the plane inside the triangle. We can parametrize S as $\mathbf{r}(s,t) = \langle s, t, 4-s-2t\rangle$ for $0 \le s \le 2$, $0 \le t \le 1$, and then $\nabla\times\mathbf{F} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y\rangle = \langle 1-(-1), 0-0, 0-(-2y)\rangle = \langle 2, 0, 2y\rangle$ while $\mathbf{n} = (d\mathbf{r}/ds)\times(d\mathbf{r}/dt) = \langle 1,0,-1\rangle\times\langle 0,1,-2\rangle = \langle 1,2,1\rangle$ (correct orientation since z-coordinate is positive). Then $(\nabla\times\mathbf{F})\cdot\mathbf{n} = 2+2y = 2+2t$, and so the surface integral is $\int_0^2\int_0^1(2+2t)\,dt\,ds = \boxed{6}$.

   (g) Use Stokes's theorem: $\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S(\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$ where S is the given portion of the surface. Parametrize S as $\mathbf{r}(s,t) = \langle s, t, s^2t\rangle$ for $0 \le s \le 1$, $0 \le t \le s$, and then $\nabla\times\mathbf{F} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y\rangle = \langle 1-0, 0-0, 2x-2x\rangle = \langle 1,0,0\rangle$ while $\mathbf{n} = (d\mathbf{r}/ds)\times(d\mathbf{r}/dt) = \langle 1,0,2st\rangle\times\langle 0,1,s^2\rangle = \langle -2st, -s^2, 1\rangle$ (correct orientation since z-coordinate is positive). Then $(\nabla\times\mathbf{F})\cdot\mathbf{n} = -2st$ so the surface integral is $\int_0^1\int_0^s -2st\,dt\,ds = \boxed{-1/4}$.

   (h) Use the divergence theorem. However, note that the surface is not closed, so we must close it and then subtract the flux through the extra plane. We close it by including the plane $z = 0$ with $0 \le x, y \le 1$. The solid is $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$ and also $\text{div}(\mathbf{F}) = 6xy + 2xy + 0 = 8xy$. Thus by the divergence theorem, the flux through the solid is $\iiint_D \text{div}(\mathbf{F})\,dV = \int_0^1\int_0^1\int_0^1 8xy\,dz\,dy\,dx = 2$. For the piece being subtracted, we have a parametrization $\mathbf{r}(s,t) = \langle s,t,0\rangle$ for $0 \le s \le 1$, $0 \le t \le 1$. Then $\mathbf{n} = (d\mathbf{r}/ds)\times(d\mathbf{r}/dt) = \langle 1,0,0\rangle\times\langle 0,1,0\rangle = \langle 0,0,1\rangle$, but this has the wrong orientation since it must point downward. Then $\mathbf{F}\cdot(-\mathbf{n}) = -8st$, and so the surface integral is $\int_0^1\int_0^1 -8st\,dt\,ds = -2$. Thus, the flux across the remaining five planes is $2-(-2) = \boxed{4}$.

   (i) Use the divergence theorem. As above the surface is not closed. Close it by including the bottom disc with $x^2+y^2 \le 1$ and $z = 0$. The solid is $0 \le r \le 1$, $0 \le \theta \le 2\pi$, $0 \le z \le 1-r^2$ in cylindrical and also $\text{div}(\mathbf{F}) = y^2 + x^2 = r^2$. Thus by the divergence theorem, the flux through the solid is $\iiint_D \text{div}(\mathbf{F})\,dV = \int_0^{2\pi}\int_0^1\int_0^{1-r^2} r^2\,r\,dz\,dr\,d\theta = \pi/6$. For the piece being subtracted, we have a parametrization $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle$ for $0 \le \theta \le 2\pi$ and $0 \le r \le 1$. Then $\mathbf{n} = (d\mathbf{r}/dr)\times(d\mathbf{r}/d\theta) = \langle\cos\theta,\sin\theta,0\rangle\times\langle -r\sin\theta, r\cos\theta, 0\rangle = \langle 0,0,r\rangle$, but this has the wrong orientation since it must point downward. Then $\mathbf{F}\cdot(-\mathbf{n}) = -r\sqrt{x^2+y^2} = r^2$, and so the surface integral is $\int_0^{2\pi}\int_0^1 -r^2\,dr\,d\theta = -2\pi/3$. The flux through the top is then $\pi/6 - (-2\pi/3) = \boxed{5\pi/6}$.

   (j) Use the divergence theorem. As above the surface is not closed. Close it by including the bottom disc with $x^2+y^2 \le 5$ and $z = 0$. The solid is $0 \le \varphi \le \pi/2$, $0 \le \theta \le 2\pi$, $0 \le \rho \le \sqrt{5}$ in spherical and also $\text{div}(\mathbf{F}) = (3x^2+2z^2) + (x^2+2z^2) + 4y^2 = 4(x^2+y^2+z^2) = 4\rho^2$. Thus by the divergence theorem, the flux through the solid is $\iiint_D \text{div}(\mathbf{F})\,dV = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{\sqrt{5}} 4\rho^2\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = 40\pi\sqrt{5}$. For the piece being subtracted, we have a parametrization $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle$ for $0 \le \theta \le 2\pi$ and $0 \le r \le \sqrt{5}$. Then $\mathbf{n} = (d\mathbf{r}/dr)\times(d\mathbf{r}/d\theta) = \langle\cos\theta,\sin\theta,0\rangle\times\langle -r\sin\theta, r\cos\theta, 0\rangle = \langle 0,0,r\rangle$, but this has the wrong orientation since it must point downward. Then $\mathbf{F}\cdot(-\mathbf{n}) = 0$, so the flux through the bottom is zero. Therefore, the flux through the top piece is simply $\boxed{40\pi\sqrt{5}}$.