Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													
out of	8	9	8	9	8	8	8	8	9	9	8	8	100

MATH 2321 Final Exam

December 17, 2020

Instructor's name________________________ Your name________________________

Please check that you have 12 different pages.

Answers from your calculator, without supporting work, are worth zero points.

1. Consider the function $f(x, y, z) = x^3\sqrt{y^2 + z^2}$.

(a) (3 points) Find the gradient ∇f of the function f.

(b) (3 points) Find the linearization of the function f at the point (2,3,4).

(c) (2 points) Use the linearization of f from part (b) to estimate the number $(1.98)^3\sqrt{(3.02)^2 + (4.01)^2}$.

2. Let $f(x, y, z) = \ln(x^2 + 2y^2 + yz)$.

(a) (3 points) Find the rate of change of f with respect to z at the point $(1, 1, 2)$.

(b) (3 points) Find the minimum rate of change of f at the point $(1, 1, 2)$ **and** the direction in which this minimum rate of change occurs.

(c) (3 points) Find a direction in which the value of f is not changing at the point $(1, 1, 2)$.

3. Consider the real valued function $F(x,y) = xe^{x-3} - y^3$.

(a) (4 points) Find an equation of the tangent line to the **level curve** $F(x,y) = 2$ at the point $(3,1)$.

(b) (4 points) Find an equation of the tangent plane to the **graph of the function** F, with equation $z = F(x,y)$, at the point $(3,1,2)$.

4. Consider the function $f(x, y) = -3x^2 + 3xy - y^3$.

(a) (3 points) This function has **two** critical points. Find them both.

(b) (6 points) Classify **both** critical points of the function f as local maximum, local minimum, or saddle points.

5. (8 points) Using the **Lagrange Multiplier Method**, find the global minimum of the function

$$f(x, y, z) = x^2 + 3y^2 + z^2$$

on the plane with equation

$$x + 4y + 5z = 6.$$

No credit is given for using a different method than the Lagrange Multiplier method.

6. Consider the plane region R bounded above by $y = 12 - x$ and $y = 5x$ and below by $y = x^2$.

(a) (2 points) Sketch the region R by labeling carefully all the boundaries.

(b) (6 points) Integrate $f(x, y) = 2x$ over the region R.

7. (8 points) Evaluate the integral

$$\int_0^1 \int_x^{\sqrt{2-x^2}} y \, dydx,$$

by converting it to **polar coordinates**.
(Note: No points will be given, if polar coordinates are NOT used.)

8. (8 points) Evaluate

$$\iiint_E z\, dV,$$

where E lies in the first octant ($x \geq 0,\ y \geq 0,\ z \geq 0$) between the spheres with equations

$$x^2 + y^2 + z^2 = 1 \quad \text{and} \quad x^2 + y^2 + z^2 = 4.$$

9. (9 points) Find the mass of the solid under the paraboloid $z = x^2 + y^2$ and above the disc in the xy-plane where $x^2 + y^2 \leq 3$, with the density

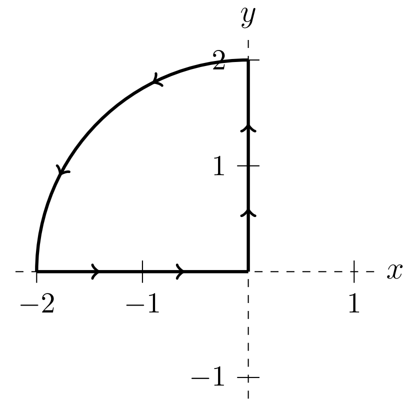
$$\delta(x, y, z) = 1 + x^2 + y^2 \text{ kg/m}^3.$$

Your answer should include **units**.

10. (9 points)

Calculate the line integral of

$$\boldsymbol{F}(x, y) = (x - y^3, x^3 + y^5)$$



along the curve $C = C_1 + C_2 + C_3$, where
C_1 is the line segment from $(0, 0)$ to $(0, 2)$,
C_2 is a quarter-circle from $(0, 2)$ to $(-2, 0)$,
C_3 is a line segment from $(-2, 0)$ to $(0, 0)$, as shown in the picture to the right.

11. (8 points) Evaluate the flux integral $\iint_M \boldsymbol{F} \cdot \boldsymbol{n}\, dS$ where

$$\boldsymbol{F}(x, y, z) = \big(x, y, 3\big)$$

and M is the surface that consists of the cylinder

$$\{x^2 + y^2 = 4,\ 0 \le z \le 2\}$$

plus the disk

$$\{x^2 + y^2 \le 4,\ z = 0\}$$

on the bottom of this cylinder. The surface M is oriented by the outward normal.

12. Consider the vector field $\boldsymbol{F}(x, y, z) = \big(2, xz, z^3\big)$.

(a) (3 points) Calculate the curl of $\boldsymbol{F}$.

(b) (5 points) Using Stokes' Theorem evaluate the line integral $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$, where C is the boundary of the surface $z = xy^2$, $0 \leq x, y \leq 1$, oriented counterclockwise as viewed from above.