Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													
out of	9	9	9	9	9	9	9	9	7	7	7	7	100

Math 2321 Final Exam

December 12, 2017

Instructor's name____________________________ Your name____________________________

Please check that you have 12 different pages.

Answers from your calculator, without supporting work, are worth zero points.

1) (9 points) Suppose that the xy-plane is occupied by a charged metal plate. Let $E = E(x, y)$ be the electric charge, in coulombs, of the plate at the point (x, y) meters. Suppose that

$$\frac{\partial E}{\partial x}\Big|_{(1,2)} = 3 \text{ C/m} \quad \text{and} \quad \frac{\partial E}{\partial y}\Big|_{(1,2)} = -2 \text{ C/m}.$$

A particle is moving along the plate so that it position, at time t seconds, is $(x, y) = (t^2, t^3 + t)$. Find the rate of change of the electric charge at the particle's location, in C/s, at time $t = 1$ second.

2) (9 points) The air temperature in a room at a point (x, y, z) is given by

$$T(x, y, z) = z(x-1)^2 + y^3 \quad {}^\circ\mathrm{C},$$

where x, y, z are measured in meters. In which direction (as a unit vector in $\mathbb{R}^3$) should a bug at the point $(2, 1, 1)$ fly in order to cool off the fastest, and at what rate (in degrees Celsius per meter) will the air temperature around it change if it moves in that direction?

3) (9 points) Consider the surface M given by $(x-1)^2 + y^3 - z = 0$. At which points on M is the tangent plane parallel to the plane given by $4x - 27y + z = 2$?

4) (9 points) Consider the surface M given by the parametrization $\mathbf{r}(u,v) = (u^3, v^3, uv)$. Give a parametric equation for the tangent plane to M at the point where $(u,v) = (1,-1)$. Also calculate a normal vector to M at this point.

5) (9 points) Consider the function

$$f(x, y) = xy - x - y + 1$$

on the set E of all points (x, y) such that $x \geq 0,\ y \geq 0$ and $x + y \leq 4$. Find the global maximum and minimum values of the function f on E and determine the points at which these values are attained.

6) (9 points) Evaluate the iterated integral

$$\int_0^2 \int_{x\sqrt{3}}^{\sqrt{16-x^2}} \sin\left(x^2 + y^2\right) \, dy \, dx \,.$$

7) (9 points) Consider the solid region T between the planes given by $x + 2y - z = 0$ and $3x + 4y - z + 2 = 0$ above the filled-in triangle in the xy-plane which has vertices given by $(0, 0, 0)$, $(1, 0, 0)$ and $(0, 1, 0)$. Suppose that

$$f(x, y, z) = \frac{1}{1 + x + y}.$$

Calculate $\iiint_T f(x, y, z)\, dV$. (Hint: If you do this right, all of the integrals are very easy.)

8) (9 points) Let T be the solid region which is inside the sphere given by $x^2 + y^2 + z^2 = 4$ and above the plane where $z = 1$. Assume that x, y, and z are measured in meters. Suppose the density of the solid region S is given by

$$\delta(x, y, z) = \frac{z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \ \text{kg/m}^3.$$

Find the total mass of the solid region T.

9) Let $f(x, y) = xy^2 - x^2$.

a) (2 points) Let $\mathbf{F}(x, y) = \vec{\nabla} f(x, y)$. Calculate the vector field $\mathbf{F}(x, y)$.

b) (5 points) Let $\mathbf{F} = \mathbf{F}(x, y)$ be the vector field from part a). Let $C = C_1 + C_2$ be the oriented curve where C_1 is the upper half of the circle of radius 5, center at the origin, oriented counterclockwise, and C_2 is the oriented line segment from $(-5, 0)$ to $(2, -1)$. Calculate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

10) (7 points) Let $\mathbf{F}(x, y) = (1 - y, -2x + e^{y^2})$. Let C be the oriented curve which starts at $(0, 0)$, goes along a straight line to $(1, 6)$, then goes along a straight line to $(2, 6)$, and finally goes along a straight line to $(1, 0)$. (So that C is three sides of a parallelogram.) Calculate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

11) (7 points) Let T be the solid right circular cylinder where $x^2 + y^2 \leq 9$ and $0 \leq z \leq 5$. Let M be the boundary of T, oriented outward. Find the flux of the vector field

$$\mathbf{F}(x, y, z) = \left(7x + xy^2,\, \cos x + e^z,\, -zy^2\right)$$

through M.

12) Let $\mathbf{F} = \left(e^{z^2} - y,\, e^{z^3} + x,\, \cos(xz)\right)$, and let M be the upper hemisphere of the sphere of radius 1, centered at the origin, with the upward orientation.

a) (2 points) Compute $\operatorname{curl} \mathbf{F}$.

b) (5 points) Compute the flux of $\operatorname{curl} \mathbf{F}$ through M.