Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													

Math 2321 Final Exam

December 12, 2014

Instructor's name____________________________ Your name____________________________

Please check that you have 10 different pages.

Answers from your calculator, without supporting work, are worth zero points.

1) (3 points each) Consider the surface M in $\mathbb{R}^3$ where $z = x^2 + y^2$.

a) Find an equation for the tangent plane to the surface at the point $(2, 1, 5)$.

b) Give a vector equation, or parametric equations, for the line which passes through the point $(2, 1, 5)$ and is normal to the surface M at $(2, 1, 5)$.

c) Determine whether or not the point $(1, -3, 2)$ is on your line from part b). You must show your supporting work.

2) (4 points each) The wind-chill index $W = W(T, v)$ is the perceived temperature when the actual temperature is T and the wind speed is v. Suppose that W and T are measured in °C and v is measured in km/h. Assume that $W(-20, 50) = -35$ and $W(-20.5, 50) = -35.6$,

a) Use the data to estimate $\partial W/\partial T$ at the point $(T, v) = (-20, 50)$.

b) Consider $\partial W/\partial v$ at the point $(T, v) = (-20, 50)$. Intuitively/physically, should this be positive, negative, or zero? Explain.

3) (9 points) Suppose that $R = R(u, v, w)$ and that $\vec{\nabla} R(3, 1, 2) = \frac{1}{7}(3, 1, 2)$. Also suppose that $u = x + 2y$, $v = 2x - y$, and $w = 2xy$. Find R_x and R_y when $x = y = 1$.

4) (9 points) Suppose that electric charge is distributed on a metal plate in such a way that the charge in coulombs, at a point (x, y), in meters, is given by $Q(x, y) = \sin(xy)$. At the point $(3, 0)$, find the maximum rate of change (in coulombs/meter) of the charge and the direction in which it occurs.

5) (3 points each) Let $\mathbf{r}(u,\ v) = (u^2 + v^2,\ u, v)$ be a parametrization of a surface S in $\mathbb{R}^3$.

a) Give an equation for S in the form $f(x,\ y,\ z) = 0$ (i.e., describe S as a level surface) and sketch the surface S.

b) Compute the tangent vectors $\mathbf{r}_u(1, 2)$ and $\mathbf{r}_v(1, 2)$.

c) Give a parameterization of the tangent plane of S at $\mathbf{r}(1, 2)$.

6) (10 points) Use Lagrange multipliers to find the five critical points of the function $f(x, y, z) = xz - y^2$ restricted to the paraboloid where $x^2 + y + z^2 = 1$ (i.e., subject to the constraint $x^2 + y + z^2 = 1$).

7) Consider the iterated integral $\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \sqrt{y}\, dxdy$ and let $\int\int_R \sqrt{y}\, dA$ be the corresponding double integral.

a) (4 points) Sketch the region of integration R.

b) (5 points) Give an iterated integral for $\int\int_R \sqrt{y}\, dA$ in which the order of integration is reversed. **Do not evaluate this integral.**

8) (9 points) Consider a solid that occupies the region in $\mathbb{R}^3$ that lies inside the (top) hemisphere where $x^2 + y^2 + z^2 = 4$ and $z \geq 0$ but lies outside the cylinder where $x^2 + y^2 = 3$. Assume that x, y, and z are measured in meters. Suppose its density is given by $\delta(x, y, z) = \frac{1}{x^2+y^2+z^2}$ kg/m^3. Find the total mass of the solid.

9) (7 points) A particle travels through the force field $\mathbf{F} = \left(3+ye^{xy}, xe^{xy}+2y\sin(y^2)\right)$ Newtons along the following oriented path C: the line segment from the origin to the point (0, 1), then along the line segment from (0, 1) to the point $(2,-1)$, then along the line segment from $(2,-1)$ to the point $(2,1)$, then along the line segment from $(2,1)$ to the point $(4,-1)$, and finally, along the line segment from $(4,-1)$ to the point $(4,0)$. Assume that x and y are in meters. Calculate the work done by $\mathbf{F}$ along C.

(Hint: This can be done **without** parameterizing five different line segments.)

10) (7 points) A particle starts at the origin, moves along a straight line to the point $(0, 4)$, then moves clockwise along the circle of radius 4, centered at the origin, to the point $(-4, 0)$, where it stops; let C denote this oriented path of the particle.

While traveling, the particle moves through the force field

$$\mathbf{F} = (8xy + 4x - 6y,\ 7^{3y+1} + \sin(\sqrt{y+5}\,) + 4x^2),$$

where $\mathbf{F}$ is in Newtons, and x and y are in meters. Calculate the work done by F along C, i.e., calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

11) (7 points) Consider the vector field $\mathbf{F}(x, y, z) = (xz, yz, z^2)$. Let M be the surface of the half-cone $z = \sqrt{x^2 + y^2}$ where $z \leq 1$, oriented downward. Calculate the flux integral $\displaystyle\int\!\!\int_M \mathbf{F} \cdot \mathbf{n}\, dS$ of $\mathbf{F}$ through M.

12) Consider the vector field $\mathbf{F}(x, y, z) = (x^2z^2, y^2z^2, xyz)$.

a) (2 points) Calculate the curl, $\vec{\nabla} \times \mathbf{F}$, of $\mathbf{F}$.

b) (5 points) Let M be the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 1$, oriented upward. Compute the flux of the curl: $\int\int_M (\vec{\nabla} \times \mathbf{F}) \cdot \mathbf{n} \, dS$.