Lecture: Monday-Wednesday, 2:50pm-4:30pm, Richards Hall 160.

Instructor: Evan Dummit (he/him/his), Lake Hall 571, edummit@northeastern.edu.

Instructor Office Hours: Wednesday 4:45pm–5:45pm and Thursday 2pm–4pm, or by appointment.

Course Webpage: https://dummit.cos.northeastern.edu/teaching_fa26_4527.html.

Course Textbook: The instructor will write lecture notes for the course (in lieu of an official textbook) as the semester progresses.

Prerequisites: Math 3527 (Number Theory 1) or Math 3150 (Group Theory) or instructor's permission. Some basic abstract algebra (fundamentals of groups and rings) will be incorporated into the course as it progresses, as needed.

Course Philosophy: The goal of this course is to give an overview of some additional topics in number theory, continuing roughly where Math 3527 leaves off. The exact set of topics will depend on student and instructor interest, but the course topics will be drawn from the following list: Farey sequences, continued fractions and rational approximation, Diophantine equations, Pell's equation, transcendence, elliptic curves and the group law, elliptic curve factorization and cryptography, rational points on elliptic curves, quadratic integer rings, factorization in quadratic integer rings, class groups, Minkowski's theorems and the geometry of numbers, binary quadratic forms, the p-adic numbers and their properties, the Riemann zeta function, Dirichlet series, arithmetic functions, L-functions, the analytic class number formula, the prime number theorem, modular forms, modularity, and a brief overview of Fermat's last theorem.

Since this is an advanced undergraduate-level class, much of the learning, and many of the secondary topics, are developed through the homework assignments. The lectures will primarily focus on discussing the results and giving applications. The homework is an integral component of the course and, as such, it is expected that all students will work on it every week: the purpose is to require complex problem solving and combining multiple ideas together in novel ways as a way of solidifying the foundation created during the lectures.

Grades: Your course grade is determined by your scores on weekly homework assignments and three oral exams.

There are 12 homework assignments each worth 30 points, the lowest score of which is dropped, meaning that 330 points are available on the homework assignments.

The oral exams are each worth 50 points and are scored pass/fail. Each student is responsible for scheduling three 20-minute exams with the instructor. The intention of the oral exams is merely to ensure that students are doing their own work in the class, and will consist of a brief interactive discussion of several problems very similar to homework problems.

A total score of at least 450 will guarantee an A, 440 will be **at least** an A-, 430 will be **at least** a B+, 420 will be **at least** a B, 410 will be **at least** a B-, 400 will be **at least** a C+, and 390 will be **at least** a C.

Homework: Written assignments will be assigned weekly and due via Gradescope on Fridays by 11:59pm. **It is highly recommended to start work on the assignments early**: many problems will require substantial thought and effort to solve, even if the solution is ultimately fairly short. There is a **30-hour late period** during which assignments may still be submitted, possibly with a late penalty assessment. **The lowest assignment grade is dropped**, to provide you a cushion if an emergency arises and you cannot complete an assignment.

Written assignments should be organized carefully, neatly, and in complete sentences, with concise well-reasoned logical arguments. Cite any external resources used, and clearly label all problems. If you collaborated with any other students, write the names of all collaborators on the top of your assignment. **Failure to adhere to any of these guidelines may result in point deductions, at the grader's discretion.**

Collaboration Policy: Mathematics is fundamentally a collaborative endeavor, and discussing the course material with others is an excellent way to solidify your own understanding. However, it is critical not to outsource your learning! You cannot expect to retain knowledge if you do not solve your homework problems yourself (e.g., if you relied on other people or on technology to explain how to do the problems). **On written assignments, you may work together with other people, but you must write up your work independently**.

Course Schedule: The course and lecture notes are organized into five chapters, four of which will be covered.

Weeks 1-3: Chapter 6 $\sim$ Rational Approximation and Diophantine Equations: the Farey sequences, finite and infinite continued fractions, rational approximation, irrationality and transcendence, Pell's equation, Pythagorean triples, miscellaneous Diophantine equations.

(Skipped) Chapter 7 $\sim$ Elliptic Curves: elliptic curves, the group law, elliptic curves modulo p, elliptic curve factorization, elliptic curve cryptography, rational points on elliptic curves.

Weeks 4-8: Chapter 8 $\sim$ Quadratic Integer Rings: rings and ideals, Euclidean domains, PIDs and UFDs, quadratic fields and integer rings, ideal factorizations, applications of unique factorization.

Weeks 9-11: Chapter 9 $\sim$ The Geometry of Numbers: Minkowski's theorems, Lagrange's four-squares theorem, the ideal class group, binary quadratic forms, reduction and composition of quadratic forms.

Weeks 12-14: Chapter 10 $\sim$ Analytic Number Theory: the Riemann zeta function, Dirichlet series, Dirichlet's theorem on primes in arithmetic progressions, L-series, the analytic class number formula.

Resources and AI Policy: If you use **any** external resources (e.g., wikipedia, stackexchange, other books beyond the course text or notes, other people, etc.) you must say what results you are citing and where they are from. **If you happen to find a solution to an assigned problem online or elsewhere, it is plagiarism to present it as your own work without attribution of its source.**

Use of generative AI / large language models (e.g., ChatGPT, Copilot) or similar technology, is expressly prohibited in this course. This includes asking for hints or solutions to assigned problems, checking solutions, searching or summarizing course-related content, and outlining drafts of submitted work.

Although the course lectures and notes provide a basic introduction to the course material, you will not actually understand the ideas until you have used them to solve new problems yourself. **Using generative AI to assist with solving the homework problems thus short-circuits your learning**, because you will not have worked through the problems as fully as you should have. Additionally, while generative AI technology produces confident and authoritative answers, it in fact often makes mistakes both small and large.

The instructor reserves the right to require any student to explain any submitted work in person; failure or inability to do so satisfactorily will be considered evidence that the work is not your own. **If plagiarism or AI use is detected on an assignment, you will receive a warning and an automatic zero on the assignment. Another violation after the warning will result in an automatic F in the course.**

Attendance Policy: It is expected that you will attend every class. This course moves very fast, and it is quite possible to fall behind even if you only miss one day. If you miss class for any reason, it is highly advisable to consult the course lecture notes and to watch the recording of the lecture you missed. It is your responsibility to be aware of all information announced in class, including modifications to the course syllabus or schedule, even if you are absent.

If you will be absent from a class activity due to a religious observance or practice, or for participation in a university-sanctioned event (e.g., university athletics), it is your responsibility to inform the instructor during the first week of class and provide appropriate documentation if required.

Statement on Academic Integrity: A commitment to the principles of academic integrity is essential to the mission of Northeastern University. Academic dishonesty violates the most fundamental values of an intellectual community and undermines the achievements of the entire University. Violations of academic integrity include (but are not limited to) cheating on assignments or exams, fabrication or misrepresentation of data or other work, plagiarism, unauthorized collaboration, and facilitation of others' dishonesty. Possible sanctions include (but are not limited to) warnings, grade penalties, course failure, suspension, and expulsion.

Statement on Accommodations: Any student with a disability is encouraged to meet with or otherwise contact the instructor during the first week of classes to discuss accommodations. The student must bring a current Memorandum of Accommodations from the Office of Student Disability Services.

Statement on Inclusivity: Faculty are encouraged to address students by their preferred name and gender pronoun. If you would like to be addressed using a specific name or pronoun, please let your instructor know.

Statement on Evaluations: Students are requested to complete the TRACE evaluations at the end of the course.

Miscellaneous Disclaimer: The instructor reserves the right to change course policies, including the evaluation scheme of the course. Notice will be given in the event of any substantial changes.