1. Find the following:

   (a) $(1+i)/(3-2i)$.
   (b) $\text{Re}[(1+i)^{800}]$.
   (c) $\left|2e^{i\pi/2}+3e^{i\pi}\right|$.
   (d) All solutions to $z^3=-8i$.
   (e) All solutions to $z^2+iz+6=0$.
   (f) All solutions to $z^4=2025$.
   (g) All solutions to $e^{4z}=4$.
   (h) All solutions to $\sin z=-4i/3$.
   (i) All values of $\log(1+i)$.
   (j) All values of $\log(-1)$.
   (k) All values of i^i.
   (l) All values of $(1+i)^{-2i}$.

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2. Suppose $|z|=1$. Show that $\overline{e^{z}}=e^{1/z}$.

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3. For the regions $R_1=\{z:1<|z|<2\}$ and $R_2=\{z:1\le \text{Re}(z)\le 2, 0\le \text{Im}(z)\}$, identify whether they are (i) open, (ii) closed, (iii) bounded, (iv) connected, (v) simply connected.

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4. For each complex function, calculate its partial derivatives $\partial f/\partial z$ and $\partial f/\partial \overline{z}$, and determine whether the complex derivative f' exists on any open region R.
   (a) $f(z)=z^3+i\overline{z}$. (b) $f(z)=e^{\text{Re}(z)}$. (c) $f(z)=e^{z\,\text{Log}(z)}$.

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5. For $f(z)=|z|^2$ show that the complex derivative $f'(z)$ exists only at $z=0$, and that $f'(0)=0$.

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6. Verify that the function $f(x+iy)=e^{-2xy}\cos(x^2-y^2)+ie^{-2xy}\sin(x^2-y^2)$ is holomorphic.

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7. Find the following things relating to power series and Laurent series:

   (a) The radius and disc of convergence of $\sum_{n=0}^{\infty}3^n z^n$.
   (b) The radius and disc of convergence of $\sum_{n=0}^{\infty}\dfrac{1}{2^n}(z-i)^n$.
   (c) A power series expansion for $\dfrac{1}{1-2z}$ centered at $z=0$.
   (d) A power series expansion for $\dfrac{1}{1-2z}$ centered at $z=1$.
   (e) A Laurent series expansion for $\dfrac{\cos(2z)}{z^3}$ centered at $z=0$.
   (f) A power series centered at $z=0$ for $1/(1+z+2z^2+2z^3+2z^4)$ up to order 3.
   (g) A Laurent series centered at $z=0$ for $1/(z^3+z^4+z^5)$ up to the constant term.

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8. Suppose that $f(z)=\sum_{n=1}^{\infty}\dfrac{1}{n}z^n$. Show that $f'(z)=1/(1-z)$ for $|z|<1$.

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9. For real x,y, show that $\left|\cos(x+iy)\right|^2=\frac{1}{2}(\cos 2x+\cosh 2y)$.

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10. Calculate the following complex line integrals:

(a) $\int_\gamma \mathrm{Re}(z)\,dz$ where $\gamma(t) = t^2 + t^3 i$ for $0 \le t \le 1$.

(b) $\int_\gamma z^2\,dz$ where $\gamma(t) = 3t^3 + 6t^8 i$ for $0 \le t \le 1$.

(c) $\int_\gamma z^2\,dz$ where γ is the counterclockwise boundary of the square with vertices ± 7, $\pm 7i$.

(d) $\int_\gamma (4z^3 + 2/z)\,dz$ where γ is the line segment from 1 to $1+i$.

(e) $\int_\gamma \frac{1}{z}\,dz$ where $\gamma(t) = 3e^{it}$ for $0 \le t \le \pi/2$.

(f) $\int_\gamma \frac{1}{z}\,dz$ where $\gamma(t) = 3e^{it}$ for $0 \le t \le 2\pi$.

(g) $\int_\gamma \frac{1}{z-5}\,dz$ where $\gamma(t) = 3e^{it}$ for $0 \le t \le 2\pi$.

(h) $\int_\gamma \frac{e^z}{z}\,dz$ where γ is the counterclockwise circle $|z-1| = 5$.

(i) $\int_\gamma \frac{z^2}{z-1}\,dz$ where γ is the counterclockwise circle $|z-1| = 1$.

(j) $\int_\gamma \frac{e^z}{z-1}\,dz$ where γ is the counterclockwise circle $|z-1| = 1$.

(k) $\int_\gamma \frac{e^z + \sin(2z)}{z-1}\,dz$ where γ is the counterclockwise boundary of the square with vertices $\pm 2 \pm 2i$.

(l) $\int_\gamma \frac{\cos z}{z^5}\,dz$ where γ is the counterclockwise unit circle $|z| = 1$.

(m) $\int_\gamma \frac{\cos z}{z^5}\,dz$ where γ is the counterclockwise boundary of the square with vertices $\pm 2 \pm 2i$.

(n) $\int_\gamma \frac{e^z}{z-5}\,dz$ where γ is the counterclockwise circle $|z| = 2$.

(o) $\int_\gamma \frac{1}{z^2+4}\,dz$ where γ is the counterclockwise circle $|z-3i| = 2$.

(p) $\int_\gamma \frac{1}{z^2-5z}\,dz$ where γ is the counterclockwise boundary of the square with vertices ± 2, $\pm 2i$.

(q) $\int_\gamma \frac{1}{z^2-5z}\,dz$ where γ is the counterclockwise circle $|z| = 10$.

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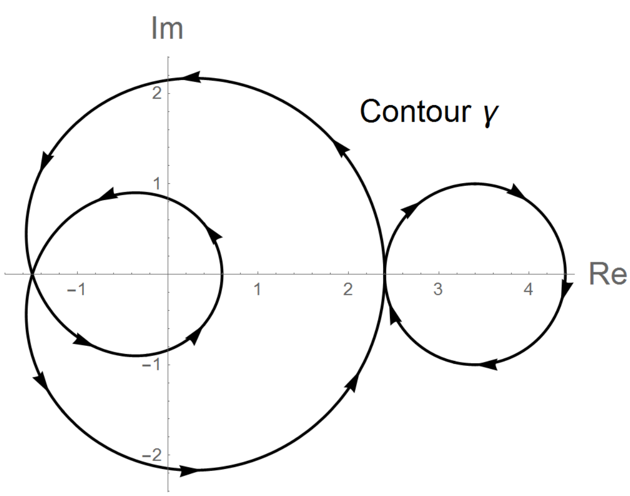
11. Let γ be the contour shown below.



(a) Find the winding numbers of γ around $0,\ 1,\ 2,\ 3,\ 4,\ i,\ 1+i,\ 2+2i$.

(b) Find $\int_\gamma \frac{1}{z}\,dz$.

(c) Find $\int_\gamma \frac{1}{z-3}\,dz$.

(d) Find $\int_\gamma \frac{e^z}{z-2}\,dz$.

(e) Find $\int_\gamma \frac{e^z}{z-4}\,dz$.

(f) Find $\int_\gamma \frac{e^z}{z^2-6z+8}\,dz$.

(g) Find $\int_\gamma \frac{\cos z}{z^3}\,dz$.

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12. Suppose that $f(z)$ is holomorphic on all of $\mathbb{C}$ and for any counterclockwise circle γ of radius 1 centered at z_0 it is true that $\int_\gamma \frac{f(z)}{(z-z_0)^2}\,dz = 0$. Show that $f(z)$ is constant.
