1. Calculate/determine the following things:

   (a) The minimum number of socks needed to be drawn from a drawer with 10 blue, 10 black, 10 red, and 11 green socks, in order to obtain at least 1 matching pair.
   (b) The minimum number of socks needed to be drawn from a drawer with 10 blue, 10 black, 10 red, and 11 green socks, in order to obtain at least 2 matching pairs.
   (c) Whether the set $(\mathbb{Z}, -)$ of integers under subtraction is a group.
   (d) The products $(sr)(sr^2)$ and $s^2r^3s^4r^5$ in the dihedral group $D_{2\cdot 10}$ of order 20.
   (e) The inverses of r^3 and sr^2 in the dihedral group $D_{2\cdot 12}$ of order 24.
   (f) The cycle decomposition of the permutation $\sigma \in S_8$ with $\sigma(n) = 9 - n$ for each $1 \le n \le 8$.
   (g) The cycle decomposition of the permutation $\sigma \in S_7$ with $\sigma(n) \equiv 3n \pmod 7$ for each $1 \le n \le 7$.
   (h) The cycle decomposition of the product $(3\,1\,4)(1\,5)$ in S_6.
   (i) The cycle decomposition of the product $(2\,7\,1\,8) \cdot (2\,8) \cdot (1\,8) \cdot (2\,8)$ in S_8.
   (j) The cycle decomposition of the inverse of $(1\,4\,2\,8\,5)(6\,7)$ in S_8.
   (k) An abelian group of order 8.
   (l) A non-abelian group of order 20.
   (m) A countably infinite abelian group.
   (n) A countably infinite non-abelian group.
   (o) An uncountably infinite abelian group.
   (p) An uncountably infinite non-abelian group.
   (q) The orders of the elements s, r, and r^2 in the dihedral group $D_{2\cdot 10}$.
   (r) The orders of the elements $(1\,2\,3)$, $(4\,5)$, and $(1\,2\,3)(4\,5)$ in S_5.
   (s) The orders of the elements $(2\,4\,5)$, $(1\,5)(2\,3)$, and $(2\,4\,5) \cdot (1\,5)(2\,3)$ in S_5.
   (t) The least upper bound in $\mathbb{R}$ of the set of rational numbers $\{0.3, 0.33, 0.333, 0.3333, 0.33333, \dots\}$.
   (u) A subset of $\mathbb{R}$ with no least upper bound.

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2. Prove the following:

   (a) Prove that if five points are selected inside a circle of radius 1, then some pair of points must be within a distance $\sqrt{2}$ of each other.
   (b) Let $f : \{1, 2, 3, \dots, n\} \to \{1, 2, 3, \dots, n\}$ be a function. Show that there exists some positive integers $A < B$ such that $f^A(i) = f^B(i)$ for each $i \in \{1, 2, 3, \dots, n\}$, where $f^n = f \circ f \circ \cdots \circ f$ composed n times.
   (c) Prove that if at least $n+1$ elements are selected from $\{1, 2, 3, \dots, 2n\}$ then some pair of them must have difference 1.
   (d) Prove that there exists a bijection between $\mathbb{Q}$ and $\mathbb{Q} \cap (0, 1)$, the set of rational numbers strictly between 0 and 1.
   (e) Prove that there exists a bijection between $(0, 1)$ and $[0, 1]$. [Hint: Cantor-Schröder-Bernstein.]
   (f) Suppose g and h are elements in a group such that $gh = hg$. Prove that $gh^n = h^n g$ for all positive integers n.
   (g) Suppose g and h are elements of a group such that $g^{-1}h^{-1} = h^{-1}g^{-1}$. Prove that $gh = hg$.
   (h) Suppose g is an element of a group G and that g has order n. Show that $g^{-1} = g^{n-1}$.
   (i) Let $f : \{1, 2, 3, \dots, n\} \to \{1, 2, 3, \dots, n\}$ be a bijection. Show that there exists some positive integer A such that $f^A(i) = i$ for each $i \in \{1, 2, 3, \dots, n\}$.
   (j) Suppose G is a group and H is a subgroup, and define the relation R on G by saying $g_1 \, R \, g_2$ whenever there exists $h \in H$ such that $g_1 = hg_2$. Prove that R is an equivalence relation.
   (k) Suppose G is an abelian group. Show that the subset $S = \{a \in g : a^2 = e\}$ is a subgroup of G.
