

1. For each matrix M , find a formula for the n th power M^n :

(a) $M = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$.

(b) $M = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$.

(c) $M = \begin{bmatrix} 5 & 6 \\ 1 & 6 \end{bmatrix}$.

(d) $M = \begin{bmatrix} 2 & -1 & 1 \\ -6 & 3 & -4 \\ -8 & 4 & -5 \end{bmatrix}$.

2. For each quadratic form, find the associated matrix and determine its definiteness:

(a) $Q(x, y) = 3x^2 - 2xy + y^2$.

(b) $Q(x, y) = -4xy - 3y^2$.

(c) $Q(x, y) = -4x^2 - 4xy - 2y^2$.

(d) $Q(x, y) = 4x^2 - 12xy + 9y^2$.

(e) $Q(x, y, z) = 7x^2 + 8xy - 4xz + 7y^2 - 4yz + 4z^2$.

(f) $Q(x, y, z) = x^2 + 4xy + 6xz + 3y^2 + 2yz + 2z^2$.

3. For each matrix M , find a singular value decomposition $M = U\Sigma V^T$ where U and V are orthogonal and Σ is a rectangular diagonal matrix:

(a) $M = \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$.

(b) $M = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$.

(c) $M = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & 6 \end{bmatrix}$.

(d) $M = \begin{bmatrix} 2 & 2 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$.

(e) $M = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \end{bmatrix}$.

(f) $M = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$.
