Lecture: Monday-Thursday, 6:00pm–7:30pm, conducted via Zoom. All lectures are recorded for later playback.

Instructor: Evan Dummit, Lake Hall 571, edummit@northeastern.edu. Office hours are via Zoom.

Course Webpage: https://web.northeastern.edu/dummit/teaching_fa20_5111.html.

Course Textbook: The instructor will write lecture notes for the course (in lieu of an official textbook) as the semester progresses. The course will generally follow the presentation in Dummit and Foote's "Abstract Algebra" (3rd edition), but it is not necessary to purchase the textbook for this course.

Prerequisites: It is suggested that you have taken (at minimum) at least one semester of undergraduate algebra, preferably more. A basic comfort level with polynomials, linear algebra, and groups will be assumed, but no specific knowledge is expected.

Course Philosophy: The goal of this course is to give an overview of groups, rings, fields, and Galois theory, as a prelude to further graduate-level work in pure mathematics. The course is designed to be complementary to Math 5112 (Algebra II), which covers commutative algebra. The primary goal of Math 5111 is to develop field theory, but because the study of fields is intimately tied to the study of polynomials and of groups, we will develop all of the necessary results as we go. The course begins with a review of polynomial rings, and then treats fields and field extensions, detours to develop some necessary group theory, and then ties everything together with Galois theory. Our end goal is, ultimately, to develop Galois theory, a fundamental and elegant relationship between fields and groups that showcases some of the beautiful interactions between different algebraic objects. As particular capstones we will discuss the formulas for the roots of cubic and quartic equations, and also prove Abel's theorem that there does not exist a similar formula for degree-5 polynomials.

Since this is a graduate-level class intended to prepare students for taking the associated qualifying exam in algebra, much of the learning, and many of the secondary topics, are developed through the homework assignments. The lectures will primarily focus on proving the results and giving basic applications. The homework is an integral component of the course and, as such, it is expected that all students will work on it every week: the purpose is to require complex problem solving and combining multiple ideas together in novel ways as a way of solidifying the foundation created during the lectures.

Grades: Your course grade consists of **1/3 homework** and **2/3 exams**.

The exam score consists of 40% midterm and 60% final.

The homework score consists of the average of the written assignment scores, with the lowest score dropped.

Course letter grades are not assigned from a fixed scale. An approximate grade range will be provided with the midterm as a guide for interpreting the score.

If you feel that an assignment or exam has been misgraded, please talk to the instructor directly. Requests for regrading will not be considered more than two days past the date the assignment or exam was returned.

Exams: There will be one take-home midterm exam, and a final exam. Depending on student and instructor preference, the final exam may be take-home, in-class, oral, or some combination of these.

The midterm is tentatively scheduled to be due on Friday, October 23rd.

The final exam will be due during the final exam week.

Homework Assignments: Written assignments will be assigned weekly and due via Canvas, typically on Fridays.

It is highly recommended to start work on the assignments early: many problems will require substantial thought and effort to solve, even if the solution is ultimately fairly short. Do not fall into the trap of only starting the assignment the evening before it is due!

The lowest assignment grade is dropped, to provide you a cushion if an emergency arises and you cannot complete an assignment. For logistical reasons, extensions cannot be granted under any circumstances.

Written assignments should be organized carefully, neatly, and in complete sentences, with concise well-reasoned logical arguments. Cite any external resources used, and clearly label all problems. All electronic submissions are expected to be easily readable. **Failure to adhere to any of these guidelines may result in point deductions, at the grader's discretion.**

Course Schedule: The course and lecture notes are tentatively organized into five chapters, as follows:

Weeks 1-2: Chapter 1 $\sim$ Polynomials and Rings: The division algorithm and Euclidean algorithm in $F[x]$, irreducibility and factorization, the Eisenstein-Schonemann criterion, modular arithmetic, rings, ideals, quotients, isomorphisms and homomorphisms.

Weeks 3-6: Chapter 2 $\sim$ Fields and Field Extensions: Fields, vector spaces, subfields, simple extensions, algebraic extensions, low-degree extensions, classical geometric constructions, splitting fields, algebraic closures, separable and inseparable extensions, transcendental extensions.

Weeks 7-9: Chapter 3 $\sim$ Groups: Groups, examples ($\mathbb{Z}/n\mathbb{Z}$, $D_{2\cdot n}$, Q_8, S_n, A_n), subgroups, orders, generation and presentations, cyclic groups, isomorphisms and homomorphisms, group actions, cosets, Lagrange's theorem, normal subgroups, quotient groups, groups acting on themselves, the class equation, abelian groups, Sylow's theorems, semidirect products.

Weeks 10-11: Chapter 4 $\sim$ Galois Theory: Field automorphisms, normal extensions, the fundamental theorem of Galois theory, characterizations of Galois extensions.

Weeks 12-14: Chapter 5 $\sim$ Applications of Galois Theory: Finite fields, the primitive element theorem, composite extensions, cyclotomic and abelian extensions, Galois groups of polynomials, solutions to cubic and quartic equations, computing Galois groups over $\mathbb{Q}$, Abel's theorem on the insolvability of the quintic.

Collaboration Policy: Mathematics is fundamentally a collaborative endeavor, and discussing the course material with others is an excellent way to solidify your own understanding. However, it is critical not to outsource your learning! You cannot expect to retain knowledge if you do not solve your homework problems yourself, whether because you relied on other people to explain to you how to do the problems, or because you relied too heavily on technological assistance.

On written assignments, you may work together with other people, but you must write up your work independently. If you use **any** external resources (e.g., wikipedia, stackexchange, other books beyond the course text or notes, other people, etc.) you must say **what results you are citing and where they are from**. If you happen to find a solution to an assigned problem online or elsewhere, it is academically dishonest to copy the solution and present it as your own work.

As a final remark: 2/3 of your course grade is determined by the exams, on which collaboration is not allowed!

Attendance Policy: It is expected that you will attend every class. This course moves very fast, and it is quite possible to fall behind even if you only miss one day. If you miss class for any reason, it is highly advisable to consult the course lecture notes and to watch the recording of the lecture you missed. It is your responsibility to be aware of all information announced in class, including modifications to the course syllabus or schedule, even if you are absent.

If you will be absent from a class activity due to a religious observance or practice, or for participation in a university-sanctioned event (e.g., university athletics), it is your responsibility to inform the instructor during the first week of class and provide appropriate documentation if required.

Statement on Academic Integrity: A commitment to the principles of academic integrity is essential to the mission of Northeastern University. Academic dishonesty violates the most fundamental values of an intellectual community and undermines the achievements of the entire University. Violations of academic integrity include (but are not limited to) cheating on assignments or exams, fabrication or misrepresentation of data or other work, plagiarism, unauthorized collaboration, and facilitation of others' dishonesty. Possible sanctions include (but are not limited to) warnings, grade penalties, course failure, suspension, and expulsion.

Statement on Accommodations: Any student with a disability is encouraged to meet with or otherwise contact the instructor during the first week of classes to discuss accommodations. The student must bring a current Memorandum of Accommodations from the Office of Student Disability Services.

Statement on Classroom Behavior: Disruptive classroom behavior will not be tolerated. In general, any behavior that impedes the ability of your fellow students to learn will be viewed as disruptive.

Statement on Inclusivity: Faculty are encouraged to address students by their preferred name and gender pronoun. If you would like to be addressed using a specific name or pronoun, please let your instructor know.

Statement on Evaluations: Students are requested to complete the TRACE evaluations at the end of the course.

Miscellaneous Disclaimer: The instructor reserves the right to change course policies, including the evaluation scheme of the course. Notice will be given in the event of any substantial changes.