Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													
out of	8	8	8	9	8	8	9	9	9	8	8	8	100

Math 2321 Final Exam

April 24, 2019

Instructor's name________________________ Your name________________________

Please check that you have 12 different pages.

Answers from your calculator, without supporting work, are worth zero points.

1) Consider the surface given by the equation $z = 3x^2y + y^2\sqrt{x}$ and the point $P = (1, -2, -2)$ on this surface.

(a) (4 points) Find a standard equation of the tangent plane to the surface at the point P.

(b) (4 points) Give a unit normal vector to the tangent plane from part (a).

2) Consider the function $F(x, y, z) = x^2 + y^2 + z$.

(a) (4 points) Sketch the level set of F that contains the point $P = (1, 1, 3)$.

(b) (4 points) Give a vector equation of the line that is perpendicular to the tangent plane to the level surface from (a) through point P.

3) Consider the function $f(x, y) = e^{2(y-1)}\sqrt{x}$.

(a) (3 points) Starting at $(4, 1)$, in what direction does f increase most rapidly with respect to distance? Give your answer as a unit vector.

(b) (2 points) Determine the maximum value of the instantaneous rate of change of the function f at $(4, 1)$ with respect to distance.

(c) (3 points) Calculate the instantaneous rate of change of f at $(4, 1)$ in the direction of $(-3, -4)$ with respect to distance.

4) Consider the function $f(x, y) = x^3 + y^3 + 3xy + 3$.

(a) (5 points) Find all critical points of the function f.

(b) (4 points) Classify the critical points as local minimum, local maximum, or saddle points.

5) (8 points) **Using the Lagrange Multiplier Method**, find the point P on the surface $12x + 4y + 3z = 169$ closest to the origin $O = (0, 0, 0)$. (**Hint**: Minimize the square distance from the point P to the point O).

No credit is given for using a different method than the Lagrange Multiplier method.

6) (8 points) Evaluate the following integral by switching the order of integration:

$$\int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin(y)}{y} \, dy \, dx.$$

7) Consider the vector field $\mathbf{F} = (6x^3y^2, 3x^4y)$.

(a) (3 points) Show that $\mathbf{F}$ is a conservative vector field.

(b) (3 points) Find a function f such that $\mathbf{F} = \nabla f$. Show work!

(c) (3 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C consists of the curve $y = xe^{x^2-1} + \sin(\pi x)$ from $(0,0)$ to $(1,1)$ followed by the curve $x = \sqrt{1 + (y-1)^3}$ from $(1,1)$ to $(0,0)$.

8) (9 points) Find the surface area of the surface that is cut from the saddle-shaped surface $z = xy$ by the cylinder $x^2 + y^2 = 1$.

9) (9 points) Let S be the solid region in the first octant ($x \geq 0,\ y \geq 0,\ z \geq 0$) between the spheres with equations $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$. Find the **mass** of S if the density of the solid region S is given by

$$\delta(x, y, z) = \frac{z^2}{(x^2 + y^2 + z^2)^2} \ \text{kg/m}^3.$$

Assume that $x, y,$ and z are measured in meters.

10) (8 points) Let $\mathbf{F} = (\sin(x^3), 2ye^{x^2})$. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C consists of two line segments, which go from $(0, 0)$ to $(2, 2)$, and then from $(2, 2)$ to $(0, 2)$.

11) (8 points) Let T be the solid right circular cylinder of radius 3, centered around the z-axis, for $1 \leq z \leq 6$, where all lengths are in meters. Let M denote the boundary surface of the solid T; so that M consists of the cylindrical side together with disks on the top and bottom. We give M its default outward-pointing orientation. Compute the flux of the vector field $\mathbf{F}(x, y, z) = (xz, yz, xy)$ newtons through M.

12) Let $\mathbf{F}(x, y, z) = (-x^2yz,\, xy^2z,\, xy + z^2)$.

(a) (3 points) Calculate $\vec{\nabla} \times \mathbf{F} =$ the curl of $\mathbf{F}$.

(b) (5 points) Let M be the chopped off paraboloid given by $z = x^2 + y^2$ and $0 \leq z \leq 9$ (note that there is no closing disk at the top). Give M the upward orientation (i.e., the orientation where unit normal vectors have a positive z-component). Calculate the flux of $\vec{\nabla} \times \mathbf{F}$ through M.