Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													

Math 2321 Final Exam

May 1, 2015

Instructor's name____________________ Your name____________________

Please check that you have 10 different pages.

Answers from your calculator, without supporting work, are worth zero points.

1) Consider the function $f(x, y) = xe^{xy} + y\sin(x)$. Let $\mathbf{p}$ be the point $(2, 0)$, and let $\mathbf{v}$ be the vector from the point $(0, 1)$ to the point $(3, 5)$.

a) (5 points) Find the directional derivative $D_{\mathbf{u}}f(\mathbf{p})$, where $\mathbf{u}$ is the unit vector in the direction of $\mathbf{v}$.

b) (3 points) Find the direction, as a unit vector, in which f increases most rapidly at $\mathbf{p}$.

2) (3 points each) Consider the function $f(x, y, z) = xe^y + xz^2$

a) Find the partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ of the function f at the point $\mathbf{p} = (2, 0, 1)$.

b) Find the linearization $L(x, y, z)$ of $f(x, y, z)$ at $\mathbf{p}$.

c) Use the linearization of f to estimate the value of f at $(1.9, 0.1, 1.5)$. (The "exact" value of $f(1.9, 0.1, 1.5)$ from your calculator is worth zero points.)

3) (7 points) Consider the function $f(x, y) = x^2 - y^2 + 1$. On a single copy of the xy-plane sketch the levels curves, where $f = 0$, $f = 1$, and $f = 2$. In your sketch, indicate which level curves are which, and include the value all of the intercepts with the $x-$ and $y-$axes.

4) Consider the function $f(x, y, z) = x^2 + y^2 + \sqrt{xz}$.

a) (7 points) Find an equation of the tangent plane to the level surface of f at the point $(1, -1, 1)$.

b) (3 points) Find the coordinates of the point of intersection of the x-axis with the tangent plane from part (a).

5) (10 points) Find the points at which the function $f(x, y) = 2x^2 + y^2 + 2x^2y$ attains a local minimum value, a local maximum value, or has a saddle point.

6) (10 points) Find the global minimum and the global maximum values of the function $f(x, y) = -2 + x^2 + 2y^2$ in the closed disk D where $x^2 + y^2 \leq 2$, as well as the coordinates of the points where these extreme values are attained.

7) Consider a double integral, which is equal to the given iterated integral: $\displaystyle\iint_R (x^3 - y^2)\, dA = \int_1^{e^3} \int_0^{\ln x} (x^3 - y^2)\, dy\, dx.$

a) (3 points) Sketch the region of integration R. Label everything carefully.

b) (5 points) Reverse the order of integration from what you were originally given, write down the resulting iterated integral using the new order. DO NOT EVALUATE THIS INTEGRAL.

8) (10 points) A solid occupies the region S, which is in the 1st octant (where $x \geq 0, y \geq 0$, and $z \geq 0$) of the solid where $0 \leq z \leq 1$ and $1 \leq x^2 + y^2 \leq 4$. Suppose that x, y, z are measured in meters, and the solid has density given by $\delta(x, y, z) = \frac{5}{x^2+y^2}$ kg/m^3. Calculate the mass of the solid.

9) This problem concerns the vector field $\mathbf{F}(x, y, z) = (ye^z,\ xe^z + z,\ xye^z + y + 2z)$.

a) (2 points) Show that $\mathbf{F}$ is conservative, **without** producing a potential function for $\mathbf{F}$.

b) (4 points) Find a potential function $f(x, y, z)$ for $\mathbf{F}(x, y, z)$.

c) (2 points) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve in $\mathbb{R}^3$ consisting of straight line segments from $(0, 0, 0)$ to $(1, 1, 0)$, then to $(2, 1, 1)$, and finally to $(3, 2, 1)$.

10) Suppose $\mathbf{F} = (3y + x^2,\ x^2 + e^{-y^2})$ represents a force field in Newtons, where x and y are in meters.

a) (1 point) Compute the 2-dimensional curl of $\mathbf{F}$.

b) (5 points) Find the work done by $\mathbf{F}$ on a particle which moves along the curve C given by three sides of a square, starting from $(1, 0)$, to $(1, 1)$, then to $(0, 1)$, and finally to $(0, 0)$.

11) (7 points) Consider the vector field $\mathbf{F}(x,\, y,\, z) = (\cos z\, +\, y^2 e^z,\, xe^{-z},\, z^2)$. Let M be the part of the hemisphere where $z = \sqrt{4 - x^2 - y^2}$ that lies inside the cylinder where $x^2 + y^2 = 3$, oriented upward. Compute the flux integral $\displaystyle\int\!\!\int_M \mathbf{F}\cdot\mathbf{n}\; dS$ of $\mathbf{F}$ through M. Note that the surface M is **not** closed.

12) (7 points) Consider the vector field $\mathbf{F}(x,\, y,\, z) = (2y\cos z\,,\, e^x \sin z\,,\, e^z)$. Let M be the **top hemisphere** of the sphere of radius 3, centered at the origin, oriented upward. Compute the flux integral $\displaystyle\int\!\!\int_M (\vec{\nabla} \times \mathbf{F}) \cdot \mathbf{n}\; dS$.