

In all problems, assume each surface is oriented with “upward” or “outward” orientation. Circulation and flux along/across a curve always refer to counterclockwise circulation and outward normal flux.

1. Compute the following:

- (a) $\int_C x^2 ds$, where C is the line segment from $(0, 1)$ to $(3, 2)$.
 - (b) $\int_C x dx + y dy$, where C is the circle $x^2 + y^2 = 9$.
 - (c) The integral of $\sqrt{x^2 + y^2}$ on the portion of the helix $\mathbf{r}(t) = \langle \cos 3t, \sin 3t, 4t \rangle$ for $0 \leq t \leq \pi$.
 - (d) The average value of y on the upper half of the circle $x^2 + y^2 = 4$.
 - (e) The work done by $\mathbf{F}(x, y, z) = \langle yz, xz, x^3 \rangle$ on a particle that travels from $(0, 0, 0)$ to $(1, 1, 1)$ along the curve parametrized by $\mathbf{r}(t) = \langle t^2, t^4, t^3 \rangle$.
 - (f) The (outward normal) flux of $\mathbf{F}(x, y) = \langle y + x, y - x \rangle$ across, and the circulation of \mathbf{F} along, the path C along the upper half of $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$.
 - (g) The (outward normal) flux of $\mathbf{F}(x, y) = \langle xy, y^2 \rangle$ across, and the circulation of \mathbf{F} along, the curve $\mathbf{r}(t) = \langle t^2, t^3 \rangle$ for $0 \leq t \leq 1$.
-

2. Find an equation for the tangent plane to each surface at the given point:

- (a) The surface parametrized by $\mathbf{r}(s, t) = \langle s + t, s^2 + t^2, s^3 + t^3 \rangle$ where $s = 1$ and $t = 2$.
 - (b) The surface parametrized by $\mathbf{r}(s, t) = \langle s^2, 2st, 3t^3 \rangle$ at the point $(x, y, z) = (1, 2, -3)$.
-

3. Parametrize each surface, and then set up (do not evaluate) an iterated double integral for each of the following:

- (a) $\iint_S (x^2 + y^2) d\sigma$ where S is the portion of the plane $z = 2x + 2y$ above the rectangle with $1 \leq x \leq 2$, $2 \leq y \leq 4$.
 - (b) $\iint_S (zx^2 + zy^2) d\sigma$ where S is the portion of the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = 4$.
 - (c) $\iint_S \sqrt{x^2 + y^2} d\sigma$ where S is the portion of the cone $z = 4\sqrt{x^2 + y^2}$ below $z = 8$.
 - (d) $\iint_S z^2 d\sigma$ where S is the unit sphere.
-

4. Set up (do not evaluate) an iterated double integral giving the area of each surface:

- (a) The portion of the plane $z = 3x + 4y + 11$ above the region with $0 \leq x \leq 1$ and $0 \leq y \leq 2$.
 - (b) The portion of the surface parametrized by $\mathbf{r}(s, t) = \langle s^2, st, t^2 \rangle$ with $0 \leq s \leq 1$ and $0 \leq t \leq 2$.
 - (c) The portion of the cone $z = 3\sqrt{x^2 + y^2}$ that lies between the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.
 - (d) The portion of the sphere $x^2 + y^2 + z^2 = 9$ that lies below the cone $z = \sqrt{3} \cdot \sqrt{x^2 + y^2}$.
-

5. For each vector field \mathbf{F} and surface S , compute the outward normal flux $\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma$ of \mathbf{F} across S :
- (a) $\mathbf{F} = \langle x^2, 2x^2, 3x^2 \rangle$, where S is the portion of the plane $x + y + z = 4$ with $0 \leq x \leq 1$ and $0 \leq y \leq 2$.
 - (b) $\mathbf{F} = \langle xz, yz, x^2 + y^2 \rangle$, where S is the portion of the cone $z = \sqrt{x^2 + y^2}$ inside the cylinder $x^2 + y^2 = 6$.
 - (c) $\mathbf{F} = \langle -xz, -yz, z^4 \rangle$, where S is the portion of the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = 3$.
 - (d) $\mathbf{F} = \langle y, -x, z \rangle$, where S is the portion of the surface $z = x^2 + y^2$ below $z = 2$.
 - (e) $\mathbf{F} = \langle x, y, z \rangle$, where S is the upper half of the unit sphere.
 - (f) $\mathbf{F} = \langle y^3, x^3, 1 \rangle$, where S is the portion of the paraboloid $z = 1 - x^2 - y^2$ above the xy -plane.
-

6. For each vector field, (i) find the divergence and curl of \mathbf{F} , and (ii) determine whether \mathbf{F} is conservative and (if so) find a potential function U :
- (a) $\mathbf{F} = \langle x^3 + xy, y^3 + xy, 0 \rangle$.
 - (b) $\mathbf{F} = \langle yz + 2x, xz + 2z, xy + 2y \rangle$.
 - (c) $\mathbf{F} = \langle x^2yz, x^2z^2, 2x^2yz \rangle$.
 - (d) $\mathbf{F} = \langle 2xyz + e^z, x^2z + 3, xe^z + x^2y + 2 \rangle$.
-

7. Evaluate $\int_C yz \, dx + xz \, dy + xy \, dz$, where C is the curve $\mathbf{r}(t) = \langle te^t, \arctan(t), \ln(1+t) \rangle$ for $0 \leq t \leq 1$.
-

8. Let C be the curve that runs once counterclockwise around the boundary of the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, and $(0, 1)$. Find $\oint_C (x^2 + y) \, dx + (2xy^2 - xy) \, dy$.
-

9. For the given C and \mathbf{F} , find the counterclockwise circulation $\oint_C \mathbf{F} \cdot \mathbf{T} \, ds$ and the outward normal flux $\oint_C \mathbf{F} \cdot \mathbf{N} \, ds$.
- (a) $\mathbf{F} = \langle xy^2, y^3 \rangle$, C is the boundary of the rectangle with vertices $(0, 0)$, $(2, 0)$, $(2, 3)$, $(0, 3)$.
 - (b) $\mathbf{F} = \langle 6xy, 0 \rangle$, C is the boundary of the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 2)$.
 - (c) $\mathbf{F} = \langle 2x + 3y, 4x + 5y \rangle$, C is the unit circle.
 - (d) $\mathbf{F} = \langle x^3 - y^3, x^3 + y^3 \rangle$, C is the boundary of the quarter-disc $x^2 + y^2 \leq 16$ in the first quadrant.
-

10. Let $C = C_1 \cup C_2 \cup C_3$, where C_1 is the line segment from $(-2, 0)$ to $(0, 0)$, C_2 is the line segment from $(0, 0)$ to $(\sqrt{2}, \sqrt{2})$, and C_3 is the shorter arc of the circle $x^2 + y^2 = 4$ from $(\sqrt{2}, \sqrt{2})$ to $(-2, 0)$.
- (a) Find the outward flux of $\mathbf{F} = \langle 2x^3y^2 + y^3, x^2 - 2x^2y^3 \rangle$ around C .
 - (b) Find the counterclockwise circulation of $\mathbf{F} = \langle x^2 - y^3, x^3 + y^4 \rangle$ around C .
 - (c) Find the work done by $\mathbf{F} = \langle -2y, 2x \rangle$ on a particle that travels once around C .
-