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1. (a) Take $\mathbf{r}(t) = \langle 3t, 1+t\rangle$ for $0 \le t \le 1$ with $ds = ||\mathbf{v}(t)||\, dt = \sqrt{10}\, dt$. Integral is $\int_0^1 (3t)^2\sqrt{10}\, dt = \boxed{3\sqrt{10}}$.
   (b) Take $\mathbf{r}(t) = \langle 3\cos t, 3\sin t\rangle$ for $0 \le t \le 2\pi$ with $dx = -3\sin t\, dt$ and $dy = 3\cos t\, dt$. Integral is $\int_0^{2\pi} (3\cos t)(-3\sin t)dt + (3\sin t)(3\cos t)\, dt = \int_0^{2\pi} 0\, dt = \boxed{0}$.
   (c) Have $ds = ||\mathbf{v}(t)|| = ||\langle -3\sin 3t,\, 3\cos 3t, 4\rangle|| = 5$ and $\sqrt{x^2+y^2} = 1$. Integral is $\int_0^\pi 1\cdot 5\, dt = \boxed{5\pi}$.
   (d) Average value is $\left[\int_C y\, ds\right] / \left[\int_C 1\, ds\right]$. Parametrize by $\mathbf{r}(t) = \langle 2\cos t,\, 2\sin t\rangle$ for $0 \le t \le \pi$ with $ds = ||\mathbf{v}(t)||\, dt = 2\, dt$. Numerator is $\int_0^\pi 2\sin t \cdot 2\, dt = 8$, denominator is $\int_0^\pi 2\, dt = 2\pi$, ratio is $\boxed{8/(2\pi) = 4/\pi}$.
   (e) Work is $\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C P\, dx + Q\, dy + R\, dz$. With $x = t^2$, $y = t^4$, $z = t^3$ get $dx = 2t\, dt$, $dy = 4t^3\, dt$, $dz = 3t^2\, dt$, $P = yz = t^7$, $Q = xz = t^5$, $R = x^3 = t^6$. Integral is $\int_0^1 t^7 \cdot 2t\, dt + t^5 \cdot 4t^3\, dt + t^6 \cdot 3t^2\, dt = \int_0^1 9t^8\, dt = \boxed{1}$.
   (f) Parametrize by $\mathbf{r}(t) = \langle \cos t, \sin t\rangle$ for $0 \le t \le \pi$. With $x = \cos t$, $y = \sin t$, get $dx = -\sin t\, dt$, $dy = \cos t\, dt$, $P = y + x = \sin t + \cos t$, $Q = y - x = \sin t - \cos t$.
   Circulation is $\int_C P\, dx + Q\, dy = \int_0^\pi (\sin t + \cos t)(-\sin t\, dt) + (\sin t - \cos t)(\cos t\, dt) = \int_0^\pi -1\, dt = \boxed{-\pi}$.
   Flux is $\int_C -Q\, dx + P\, dy = \int_0^\pi (\cos t - \sin t)(-\sin t\, dt) + (\sin t + \cos t)(\cos t\, dt) = \int_0^\pi 1\, dt = \boxed{\pi}$.
   (g) With $x = t^2$, $y = t^3$ get $dx = 2t\, dt$, $dy = 3t^2\, dt$, $P = xy = t^5$, $Q = y^2 = t^6$.
   Circulation is $\int_C P\, dx + Q\, dy = \int_0^1 t^5(2t\, dt) + t^6(3t^2\, dt) = \int_0^1 (2t^6 + 3t^8)\, dt = \boxed{13/21}$.
   Flux is $\int_C -Q\, dx + P\, dy = \int_0^1 -t^6(2t\, dt) + t^5(3t^2\, dt) = \int_0^1 t^7\, dt = \boxed{1/8}$.

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2. (a) Normal vector is $\mathbf{n} = (d\mathbf{r}/ds)\times(d\mathbf{r}/dt) = \langle 1, 2s, 3s^2\rangle \times \langle 1, 2t, 3t^2\rangle$. With $s = 1$ and $t = 2$ this is $\mathbf{n} = \langle 1, 2, 3\rangle \times \langle 1, 4, 12\rangle = \langle 12, -9, 2\rangle$. Point is $\mathbf{r}(1, 2) = \langle 3, 5, 9\rangle$, so equation is $\boxed{12(x-3) - 9(y-5) + 2(z-9) = 0}$.
   (b) If $(x, y, z) = (1, 2, -3)$ then $t = -1$ and $s = -1$. Then $\mathbf{n} = (d\mathbf{r}/ds)\times(d\mathbf{r}/dt) = \langle 2s, 2t, 0\rangle \times \langle 0, 2s, 9t^2\rangle$, so $s = -1$ and $t = -1$ give $\mathbf{n} = \langle -2, -2, 0\rangle \times \langle 0, -2, 9\rangle = \langle -18, 18, 4\rangle$. Equation is $\boxed{-18(x-1) + 18(y-2) + 4(z+3) = 0}$.

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3. (a) Parametrize by $\mathbf{r}(s,t) = \langle s, t, 2s+2t\rangle$ for $1 \le s \le 2$, $2 \le t \le 4$. Then $\frac{\partial\mathbf{r}}{\partial s} \times \frac{\partial\mathbf{r}}{\partial t} = \langle -2, -2, 1\rangle$ so $d\sigma = \left|\left|\frac{\partial\mathbf{r}}{\partial s} \times \frac{\partial\mathbf{r}}{\partial t}\right|\right|\, ds\, dt = 3\, ds\, dt$. Surface integral is $\int_2^4\int_1^2 (s^2+t^2)\cdot 3\, ds\, dt$.
   (b) Parametrize by $\mathbf{r}(z,\theta) = \langle 2\cos\theta,\, 2\sin\theta,\, z\rangle$ for $0 \le \theta \le 2\pi$, $0 \le z \le 4$. Then $\frac{\partial\mathbf{r}}{\partial z} \times \frac{\partial\mathbf{r}}{\partial \theta} = \langle -2\cos\theta,\, 2\sin\theta, 0\rangle$ so $d\sigma = \left|\left|\frac{\partial\mathbf{r}}{\partial z} \times \frac{\partial\mathbf{r}}{\partial \theta}\right|\right|\, dz\, d\theta = 2\, dz\, d\theta$. Surface integral is $\int_0^{2\pi}\int_0^4 8z\, dz\, d\theta$.
   (c) Parametrize by $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, 4r\rangle$ for $0 \le \theta \le 2\pi$, $0 \le r \le 2$. Then $\frac{\partial\mathbf{r}}{\partial r} \times \frac{\partial\mathbf{r}}{\partial \theta} = \langle -4r\cos\theta,\, -4r\sin\theta, r\rangle$ so $d\sigma = \left|\left|\frac{\partial\mathbf{r}}{\partial r} \times \frac{\partial\mathbf{r}}{\partial \theta}\right|\right|\, dr\, d\theta = r\sqrt{17}\, dr\, d\theta$. Surface integral is $\int_0^{2\pi}\int_0^2 r\cdot r\sqrt{17}\, dr\, d\theta$.
   (d) Parametrize by $\mathbf{r}(\theta,\varphi) = \langle \cos\theta\sin\varphi, \sin\theta\sin\varphi, \cos\varphi\rangle$ for $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi$. Then $\frac{\partial\mathbf{r}}{\partial \theta} \times \frac{\partial\mathbf{r}}{\partial \varphi} = \langle -\cos\theta\sin^2\varphi, -\sin\theta\sin^2\varphi, -\sin\varphi\cos\varphi\rangle$ so $d\sigma = \left|\left|\frac{\partial\mathbf{r}}{\partial \theta} \times \frac{\partial\mathbf{r}}{\partial \varphi}\right|\right|\, d\varphi\, d\theta = \sin\varphi\, d\varphi\, d\theta$. Surface integral is then $\int_0^{2\pi}\int_0^\pi \cos^2\varphi\sin\varphi\, d\varphi\, d\theta$.

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4. (a) Parametrize by $\mathbf{r}(s,t) = \langle s, t, 3s+4t+11\rangle$ for $0 \le s \le 1$, $0 \le t \le 2$. Then $\frac{\partial\mathbf{r}}{\partial s} \times \frac{\partial\mathbf{r}}{\partial t} = \langle -3, -4, 1\rangle$ so $d\sigma = \left|\left|\frac{\partial\mathbf{r}}{\partial s} \times \frac{\partial\mathbf{r}}{\partial t}\right|\right|\, ds\, dt = \sqrt{26}\, ds\, dt$. Surface area integral is $\int_0^1\int_0^2 1\cdot\sqrt{26}\, ds\, dt$.
   (b) Here $\frac{\partial\mathbf{r}}{\partial s} \times \frac{\partial\mathbf{r}}{\partial t} = \langle 2s, t, 0\rangle \times \langle 0, s, 2t\rangle = \langle 2t^2, -4st, 2s^2\rangle$ so $d\sigma = \left|\left|\frac{\partial\mathbf{r}}{\partial s} \times \frac{\partial\mathbf{r}}{\partial t}\right|\right|\, ds\, dt = 2\sqrt{s^4 + 4s^2t^2 + t^4}\, ds\, dt$. Surface area integral is $\int_0^1\int_0^2 1\cdot 2\sqrt{s^4+4s^2t^2+t^4}\, ds\, dt$.
   (c) Parametrize by $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, 3r\rangle$ for $0 \le \theta \le 2\pi$, $2 \le r \le 3$. Then $\frac{\partial\mathbf{r}}{\partial r} \times \frac{\partial\mathbf{r}}{\partial \theta} = \langle -3r\cos\theta,\, 3r\sin\theta, r\rangle$ so $d\sigma = \left|\left|\frac{\partial\mathbf{r}}{\partial r} \times \frac{\partial\mathbf{r}}{\partial \theta}\right|\right|\, dr\, d\theta = r\sqrt{10}\, dr\, d\theta$. Surface area integral is $\int_0^{2\pi}\int_2^3 1\cdot r\sqrt{10}\, dr\, d\theta$.
   (d) Cone is $\varphi = \pi/6$, so can parametrize by $\mathbf{r}(\theta,\varphi) = \langle 3\cos\theta\sin\varphi, 3\sin\theta\sin\varphi, 3\cos\varphi\rangle$ for $0 \le \theta \le 2\pi$, $\pi/6 \le \varphi \le \pi$. Then $\frac{\partial\mathbf{r}}{\partial \theta} \times \frac{\partial\mathbf{r}}{\partial \varphi} = \langle -9\cos\theta\sin^2\varphi, -9\sin\theta\sin^2\varphi, -9\sin\varphi\cos\varphi\rangle$ so $d\sigma = \left|\left|\frac{\partial\mathbf{r}}{\partial \theta} \times \frac{\partial\mathbf{r}}{\partial \varphi}\right|\right|\, d\varphi\, d\theta = 9\sin\varphi\, d\varphi\, d\theta$. Surface area integral is $\int_0^{2\pi}\int_{\pi/6}^\pi 1\cdot 9\sin\varphi\, d\varphi\, d\theta$.

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5. (a) Parametrize as $\mathbf{r}(s,t) = \langle s, t, 4-s-t\rangle$ for $0 \le s \le 1$, $0 \le t \le 2$. Then $\frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} = \langle 1,1,1\rangle$ and $\mathbf{F} = \langle s^2, 2s^2, 3s^2\rangle$. Surface integral is $\int_0^2 \int_0^1 \langle 1,1,1\rangle \cdot \langle s^2, 2s^2, 3s^2\rangle \, ds\, dt = \int_0^2 \int_0^1 6s^2 \, ds\, dt = \boxed{4}$.

(b) Parametrize as $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, r\rangle$ for $0 \le r \le \sqrt{6}$, $0 \le \theta \le 2\pi$. Then $\frac{\partial \mathbf{r}}{\partial r} \times \frac{\partial \mathbf{r}}{\partial \theta} = \langle -r\cos\theta, -r\sin\theta, r\rangle$ and $\mathbf{F} = \langle -r^2\cos\theta, -r^2\sin\theta, r^2\rangle$. Surface integral is $\int_0^{2\pi} \int_0^{\sqrt{6}} \langle -r\cos\theta, -r\sin\theta, r\rangle \cdot \langle -r^2\cos\theta, -r^2\sin\theta, r^2\rangle \, dr\, d\theta = \int_0^{2\pi} \int_0^{\sqrt{6}} 2r^3 \, dr\, d\theta = \boxed{36\pi}$.

(c) Parametrize as $\mathbf{r}(\theta, z) = \langle 2\cos\theta, 2\sin\theta, z\rangle$ for $0 \le \theta \le 2\pi$, $0 \le z \le 3$. Then $\frac{\partial \mathbf{r}}{\partial \theta} \times \frac{\partial \mathbf{r}}{\partial z} = \langle 2\cos\theta, 2\sin\theta, 0\rangle$ and $\mathbf{F} = \langle 2z\cos\theta, 2z\sin\theta, z^4\rangle$. Surface integral is $\int_0^{2\pi} \int_0^3 \langle 2\cos\theta, 2\sin\theta, 0\rangle \cdot \langle 2z\cos\theta, 2z\sin\theta, z^4\rangle \, dz\, d\theta = \int_0^{2\pi} \int_0^3 4z \, dz\, d\theta = \boxed{36\pi}$.

(d) Parametrize as $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, r^2\rangle$ for $0 \le \theta \le 2\pi$, $0 \le r \le \sqrt{2}$. Then $\frac{\partial \mathbf{r}}{\partial r} \times \frac{\partial \mathbf{r}}{\partial \theta} = \langle -2r^2\cos\theta, -2r^2\sin\theta, r\rangle$ and $\mathbf{F} = \langle r\sin\theta, -r\cos\theta, r^2\rangle$. Surface integral is $\int_0^{2\pi} \int_0^{\sqrt{2}} \langle -2r^2\cos\theta, -2r^2\sin\theta, r\rangle \cdot \langle r\sin\theta, -r\cos\theta, r^2\rangle \, dr\, d\theta = \int_0^{2\pi} \int_0^{\sqrt{2}} r^3 \, dr\, d\theta = \boxed{2\pi}$.

(e) Parametrize by $\mathbf{r}(\theta, \varphi) = \langle \cos\theta\sin\varphi, \sin\theta\sin\varphi, \cos\varphi\rangle$ for $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi/2$. Then $\frac{\partial \mathbf{r}}{\partial \varphi} \times \frac{\partial \mathbf{r}}{\partial \theta} = \langle \cos\theta\sin^2\varphi, \sin\theta\sin^2\varphi, \sin\varphi\cos\varphi\rangle$ and $\mathbf{F} = \langle \cos\theta\sin\varphi, \sin\theta\sin\varphi, \cos\varphi\rangle$. Integral is then $\int_0^{2\pi} \int_0^{\pi/2} \langle \cos\theta\sin^2\varphi, \sin\theta\sin^2\varphi, \sin\varphi\cos\varphi\rangle \cdot \langle \cos\theta\sin\varphi, \sin\theta\sin\varphi, \cos\varphi\rangle \, d\varphi\, d\theta = \int_0^{2\pi} \int_0^{\pi/2} \sin\varphi \, d\varphi\, d\theta = \boxed{2\pi}$.

(f) Parametrize by $\mathbf{r}(r,\theta) = \langle r\cos\theta, r\sin\theta, 1-r^2\rangle$ for $0 \le \theta \le 2\pi$, $0 \le r \le 1$. Then $\frac{\partial \mathbf{r}}{\partial r} \times \frac{\partial \mathbf{r}}{\partial \theta} = \langle 2r^2\cos\theta, 2r^2\sin\theta, r\rangle$ and $\mathbf{F} = \langle r^3\sin^3\theta, r^3\cos^3\theta, 1\rangle$. Integral is then $\int_0^{2\pi} \int_0^1 \langle 2r^2\cos\theta, 2r^2\sin\theta, r\rangle \cdot \langle r^3\sin^3\theta, r^3\cos^3\theta, 1\rangle \, dr\, d\theta = \int_0^{2\pi} \int_0^1 (2r^5\sin\theta\cos\theta + r) \, dr\, d\theta = \boxed{\pi}$.

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6. (a) $\nabla \cdot \mathbf{F} = \boxed{3x^2 + y + 3y^2 + x}$, $\nabla \times \mathbf{F} = \boxed{\langle 0, 0, y - x\rangle}$. Since $\nabla \times \mathbf{F} \ne \mathbf{0}$, field is $\boxed{\text{not conservative}}$.

(b) $\nabla \cdot \mathbf{F} = \boxed{2}$, $\nabla \times \mathbf{F} = \boxed{\langle 0,0,0\rangle}$. Since $\nabla \times \mathbf{F} = \mathbf{0}$, field is $\boxed{\text{conservative}}$. A potential is $U = \boxed{xyz + x^2 + 2yz}$.

(c) $\nabla \cdot \mathbf{F} = \boxed{2xyz + 2x^2y}$, $\nabla \times \mathbf{F} = \boxed{\langle 0, x^2y - 4xyz, 2xz^2 - x^2z\rangle}$. Since $\nabla \times \mathbf{F} \ne \mathbf{0}$, field is $\boxed{\text{not conservative}}$.

(d) $\nabla \cdot \mathbf{F} = \boxed{10yz + xe^z}$, $\nabla \times \mathbf{F} = \boxed{\langle 0,0,0\rangle}$. Since $\nabla \times \mathbf{F} = \mathbf{0}$, field is $\boxed{\text{conservative}}$. A potential function is $U = \boxed{x^2yz + xe^z + 3y + 2z}$.

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7. Integral is work done by $\mathbf{F} = \langle yz, xz, xy\rangle$ along $C$ from $\mathbf{r}(0) = \langle 0,0,0\rangle$ to $\mathbf{r}(1) = \langle e, \pi/4, \ln(2)\rangle$. $\mathbf{F}$ is conservative with potential $U = xyz$, so work is simply $U(e, \pi/4, \ln(2)) - U(0,0,0) = \boxed{e \cdot (\pi/4) \cdot \ln(2)}$.

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8. By Green's theorem, $\oint_C P\,dx + Q\,dy = \iint_R (Q_x - P_y)\,dy\,dx = \int_0^1 \int_0^1 (2y^2 - y - 1)\,dy\,dx = \boxed{-5/6}$.

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9. By Green, circulation $\oint_C P\,dx + Q\,dy = \iint_R (Q_x - P_y)\,dA$ and flux $\oint_C -Q\,dx + P\,dy = \iint_R (P_x + Q_y)\,dA$.

(a) Region is $0 \le x \le 2$, $0 \le y \le 3$. Circ is $\int_0^2 \int_0^3 (-2xy)\,dy\,dx = \boxed{-18}$, flux is $\int_0^2 \int_0^3 4y^2\,dy\,dx = \boxed{72}$.

(b) Region is $0 \le x \le 1$, $0 \le y \le 2 - 2x$. Circ is $\int_0^1 \int_0^{2-2x} (-6x)\,dy\,dx = \boxed{-2}$, flux is $\int_0^1 \int_0^{2-2x} 6y\,dy\,dx = \boxed{4}$.

(c) Region is $0 \le r \le 1$, $0 \le \theta \le 2\pi$. Circ is $\int_0^{2\pi} \int_0^1 1 \cdot r\,dr\,d\theta = \boxed{\pi}$, flux is $\int_0^{2\pi} \int_0^1 7 \cdot r\,dr\,d\theta = \boxed{7\pi}$.

(d) Region is $0 \le r \le 4$, $0 \le \theta \le \pi/2$. Circ is $\int_0^{\pi/2} \int_0^4 3r^2 \cdot r\,dr\,d\theta = \boxed{96\pi}$, flux is $\int_0^{\pi/2} \int_0^4 3r^2 \cdot r\,dr\,d\theta = \boxed{96\pi}$.

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10. Note that $C$ is the counterclockwise boundary of the polar region $0 \le r \le 2$ and $\pi/4 \le \theta \le \pi$.

(a) By Green, flux of $\mathbf{F} = \langle P, Q\rangle$ is $\iint_R (P_x + Q_y)\,dA = \int_{\pi/4}^{\pi} \int_0^2 0 \cdot r\,dr\,d\theta = \boxed{0}$.

(b) By Green, circulation of $\mathbf{F} = \langle P, Q\rangle$ is $\iint_R (Q_x - P_y)\,dA = \int_{\pi/4}^{\pi} \int_0^2 3r^2 \cdot r\,dr\,d\theta = \boxed{9\pi}$.

(c) We can use the tangential form of Green's theorem for the work integral, since it is the same as the circulation integral. So the work is $\iint_R (Q_x - P_y)\,dA = \int_{\pi/4}^{\pi} \int_0^2 4 \cdot r\,dr\,d\theta = \boxed{6\pi}$.

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