1. (a) Point list is $(1,1)$, $(0,0)$, $(2,0)$, $(2,4/3)$, $(2,4)$, $(4/9,8/9)$. Min of -2 at $(1,1)$, max of 20 at $(2,4)$.
   (b) Point list is $(\sqrt{2},\sqrt{2})$, $(-\sqrt{2},-\sqrt{2})$. Min of $-2\sqrt{2}$ at $(-\sqrt{2},-\sqrt{2})$, max of $2\sqrt{2}$ at $(\sqrt{2},\sqrt{2})$.
   (c) Point list is $(0,0)$, $(8,64)$, $(2,16)$. Min of -8 at $(2,16)$, max of 64 at $(8,64)$.

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2. (a) System is $1 = 2\lambda x$, $3 = 2\lambda y$, $x^2 + y^2 = 40$. Thus $x = 1/(2\lambda)$, $y = 3/(2\lambda)$, so $10/(4\lambda^2) = 40$ so $\lambda = \pm 1/4$, yielding $(x,y) = (2,6), (-2,-6)$. Min of -20 at $(-2,-6)$, max of 20 at $(2,6)$.
   (b) System is $y^2 = 2\lambda x$, $2xy = 2\lambda y$, $x^2 + y^2 = 12$. If $y = 0$ then get points $(\pm\sqrt{12}, 0)$. Otherwise, second equation gives $x = \lambda$, and then first equation gives $y^2 = 2\lambda^2$, so third equation is $3\lambda^2 = 12$ so $\lambda = \pm 2$ and $(x,y) = (\pm 2, \pm\sqrt{8})$. Min of -16 at $(-2,\pm\sqrt{8})$, max of 16 at $(2,\pm\sqrt{8})$.
   (c) System is $y = 3\lambda$, $x = \lambda$, $3x + y = 60$. Thus $6\lambda = 60$ so $\lambda = 10$ yielding $(x,y) = (10,30)$. Minimum does not exist (f goes to $-\infty$ for large x or large y), maximum is 300 at $(10,30)$.
   (d) System is $2 = 2\lambda x$, $4 = 2\lambda y$, $5 = 2\lambda z$, $x^2 + y^2 + z^2 = 1$. Thus $x = 2/(2\lambda)$, $y = 4/(2\lambda)$, $z = 5/(2\lambda)$, so $45/(4\lambda^2) = 1$ so $\lambda = \pm\sqrt{45/4}$, yielding $(x,y,z) = \pm\frac{1}{\sqrt{45}}(2,4,5)$. Min of $-\sqrt{45}$ at $-\frac{1}{\sqrt{45}}(2,4,5)$, max of $\sqrt{45}$ at $\frac{1}{\sqrt{45}}(2,4,5)$.
   (e) System is $yz = 2\lambda x$, $xz = 8\lambda y$, $xy = 32\lambda z$, $x^2 + 4y^2 + 16z^2 = 48$. If one variable is zero then so must be a second, yields points $(x,y,z) = (\pm\sqrt{48},0,0)$, $(0,\pm\sqrt{12},0)$, $(0,0,\pm\sqrt{3})$. If none are zero, divide first two equations to get $y/x = x/(4y)$ so $x/y = \pm 2$. Similarly, $x/z = \pm 4$. Yields points $(\pm 4, \pm 2, \pm 1)$ with all sign choices. Min of -8 and max of 8.

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3. We maximize $A = lw$ subject to $2l + w = 60\text{m}$. Using Lagrange gives $w = 2\lambda$, $l = \lambda$, $2l + w = 60\text{m}$ and so $4\lambda = 60$ and $\lambda = 15$. Thus $l = 15\text{m}$, $w = 30\text{m}$.

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4. (a) $\int_0^2 \int_y^{2y} xy^2\,dx\,dy = \int_0^2 \frac{3}{2}y^4\,dy = \frac{48}{5}$. (b) $\int_0^1 \int_{x^3}^{x^2} x\,dy\,dx = \int_0^1 (x^3 - x^4)\,dx = \frac{1}{20}$.

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5. (a) Integrals are $\int_0^1 \int_0^3 x^2 y\,dy\,dx$ and $\int_0^3 \int_0^1 x^2 y\,dx\,dy$.
   (b) Integrals are $\int_0^4 \int_{x^2}^{8\sqrt{x}} (x+y)\,dy\,dx$ and $\int_0^{16} \int_{y^2/64}^{\sqrt{y}} (x+y)\,dx\,dy$.
   (c) Integrals are $\int_0^1 \int_0^x x^3\,dy\,dx + \int_1^2 \int_0^{2-x} x^3\,dy\,dx$ and $\int_0^1 \int_y^{2-y} x^3\,dx\,dy$.

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6. (a) $\int_0^9 \int_{\sqrt{y}}^3 xy\,dx\,dy$. (b) $\int_1^2 \int_{\sqrt{x}}^x y^4\,dy\,dx + \int_2^4 \int_{\sqrt{x}}^2 y^4\,dy\,dx$.

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7. (a) Region is interior of a quarter-circle. Integral is $\int_\pi^{3\pi/2} \int_0^1 (r\cos\theta)\,r\,dr\,d\theta = \int_\pi^{3\pi/2} \frac{1}{3}\cos\theta\,d\theta = -\frac{1}{3}$.
   (b) Region is interior of a quarter-circle. Integral is $\int_{\pi/4}^{3\pi/4} \int_0^4 (r)\,r\,dr\,d\theta = \int_{\pi/4}^{3\pi/4} \frac{64}{3}\,d\theta = \frac{32\pi}{3}$.
   (c) Region is interior of a circle. Integral is $\int_0^{2\pi} \int_0^2 (4 - r^2)\,r\,dr\,d\theta = \int_0^{2\pi} 12\,d\theta = 24\pi$.

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8. Region is interior of a quarter-circle. Polar integral is $\int_0^{\pi/2}\int_0^1 \frac{1}{r}\cdot r\,dr\,d\theta = \int_0^{\pi/2}\int_0^1 1\,dr\,d\theta = \int_0^{\pi/2} 1\,d\theta = \pi/2$.

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9. Region is interior of a triangle. Reversed integral is $\int_0^4\int_0^{2y} \frac{e^y}{y}\,dx\,dy = \int_0^4 2e^y\,dy = 2(e^4-1)$.

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10. (a) $I = \int_0^2\int_x^{2x} 3(y^2-x^2)\,dy\,dx = \int_0^2 4x^3\,dx = 16$. (b) $I = \int_0^\pi\int_0^\pi 4\sin\varphi\,d\varphi\,d\theta = \int_0^\pi 8\,d\theta = 8\pi$.

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11. (a) Cylindrical: $\int_0^{2\pi}\int_0^1\int_0^{r\cos\theta+r\sin\theta} r\cdot r dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 r^3(\sin\theta+\cos\theta)\,dr\,d\theta = \int_0^{2\pi}\frac{1}{4}(\sin\theta+\cos\theta)\,d\theta = 0$.
   (b) Spherical: $\int_0^\pi\int_0^{\pi/2}\int_0^2 \rho\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \int_0^\pi\int_0^{\pi/2} 4\sin\varphi\,d\rho\,d\varphi\,d\theta = \int_0^{\pi/2} 4\,d\theta = 2\pi$.
   (c) Cylindrical: $\int_{-\pi/2}^{\pi/2}\int_0^3\int_{-1}^{r^2} \frac{1}{r}\cdot r\,dz\,dr\,d\theta = \int_{-\pi/2}^{\pi/2}\int_0^3 (r^2+1)\,dr\,d\theta = \int_{-\pi/2}^{\pi/2} 12\,d\theta = 12\pi$.
   (d) Spherical: $\int_0^{2\pi}\int_0^{\pi/4}\int_0^2 \frac{\rho^2\cos^2\varphi}{\rho}\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \int_0^{2\pi}\int_0^{\pi/4} 4\cos^2\varphi\sin\varphi\,d\rho\,d\varphi\,d\theta = 4\pi(4-\sqrt{2})/3$.

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12. (a) Rectangular: $\int_0^1\int_0^2\int_{x^2+y^2}^7 (x^2+y^2)\,dz\,dy\,dx$.
   (b) Rectangular: $\int_1^2\int_{-3}^3\int_{y^2}^9 xyz\,dz\,dy\,dx$.
   (c) Cylindrical: $\int_0^{2\pi}\int_0^3\int_r^3 (r^2+z^2)\,r\,dz\,dr\,d\theta$.
   (d) Cylindrical: $\int_{\pi/2}^{3\pi/2}\int_0^2\int_0^{4-r\sin\theta} zr\cdot r\,dz\,dr\,d\theta$.
   (e) Spherical: $\int_0^{2\pi}\int_{\pi/4}^{\pi}\int_0^2 \rho\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$.
   (f) Cylindrical: $\int_0^{2\pi}\int_1^{\sqrt{5}}\int_0^{5-r^2} 1\cdot r\,dz\,dr\,d\theta$.
   (g) Spherical: $\frac{1}{4\pi/3}\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 \rho^2\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$.
   (h) Rectangular: $\int_0^4\int_0^{\sqrt{4-x}}\int_0^{4-x-y^2} x\,dz\,dy\,dx$.

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13. Mass is $M = \iiint_D \rho(x,y,z)\,dV = \int_0^1\int_0^2\int_0^3 z\,dz\,dy\,dx = 9$g. Moments for center of mass are $M_x = \iiint_D x\rho(x,y,z)\,dV = \int_0^1\int_0^2\int_0^3 xz\,dz\,dy\,dx = 9/2$, $M_y = \iiint_D y\rho(x,y,z)\,dV = \int_0^1\int_0^2\int_0^3 yz\,dz\,dy\,dx = 9$, $M_y = \iiint_D z\rho(x,y,z)\,dV = \int_0^1\int_0^2\int_0^3 z^2\,dz\,dy\,dx = 6$. So center of mass is $\frac{1}{M}(M_x, M_y, M_z) = (\frac{1}{2}\text{cm}, 1\text{cm}, \frac{2}{3}\text{cm})$.
