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1. For vectors $\mathbf{v} = \langle 3, 0, -4\rangle$ and $\mathbf{w} = \langle -1, 6, 2\rangle$, compute the following:

   (a) $\mathbf{v} + 2\mathbf{w}$, $||\mathbf{v}||$, and $||\mathbf{w}||$.
   (b) $\mathbf{v} \cdot \mathbf{w}$ and $\mathbf{v} \times \mathbf{w}$.
   (c) A unit vector in the opposite direction of $\mathbf{v}$.
   (d) A vector of length 4 in the same direction as $\mathbf{w}$.
   (e) The angle between $\mathbf{v}$ and $\mathbf{w}$.
   (f) The vector projection $\text{proj}_{\mathbf{w}}\mathbf{v}$ of $\mathbf{v}$ onto $\mathbf{w}$.
   (g) The area of the parallelogram with sides $\mathbf{v}$, $\mathbf{w}$.

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2. Find an equation or a parametrization for each of the following:

   (a) The plane parallel to $x + 2y - 3z = 1$ containing the point $(2, -1, 2)$.
   (b) The line containing $(2, -1, 4)$ and $(3, 6, 2)$.
   (c) The line parallel to $\langle x, y, z\rangle = \langle 1 - 2t,\ 3 + 2t,\ 2 + 5t\rangle$ containing the point $(1, 1, 1)$.
   (d) The plane perpendicular to the line $\langle x, y, z\rangle = \langle t, 1 + 2t, 3 - 3t\rangle$ containing the origin.
   (e) The plane containing the points $(1, 0, 1)$, $(2, 1, 2)$, and $(3, 3, 5)$.
   (f) The intersection of the planes $x + y + 2z = 4$ and $2x - y - z = 5$.
   (g) The plane containing the vectors $\langle 1, 2, -1\rangle$ and $\langle 2, -1, 1\rangle$ and the point $(1, -1, 2)$.

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3. Find the distance from the point $(1, 3, 2)$ to the plane $x + 2y + z = 5$.

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4. At time t, a particle has position $\mathbf{r}(t) = \langle 3\cos(t),\ 5\sin(t),\ 4\cos(t)\rangle$. Compute the following:

   (a) The particle's velocity $\mathbf{v}(t)$.
   (b) The particle's speed $||\mathbf{v}(t)||$.
   (c) The distance traveled by the particle between $t = 0$ and $t = 1$.
   (d) The particle's acceleration $\mathbf{a}(t)$.
   (e) The unit tangent vector $\mathbf{T}(t)$.
   (f) The unit normal vector $\mathbf{N}(t)$.

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5. Find a parametrization for the tangent line to the curve $\langle x, y, z\rangle = s^2\mathbf{i} + s^3\mathbf{j} + s^4\mathbf{k}$ when $s = 1$.

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6. A potato is fired into the air at time $t = 0$s from the origin in a vacuum with initial velocity $\mathbf{v}(0) = (4\mathbf{i} + 8\mathbf{j} + 80\mathbf{k})$m/s. Assuming that the only force acting on the potato is the downward acceleration due to gravity of $\mathbf{a}(t) = -10\mathbf{k}$m/s², find the following:

   (a) The velocity of the potato at time t.
   (b) The position of the potato at time t.
   (c) The total time that the potato is in the air.
   (d) The potato's speed when it hits the ground.

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7. If $f(x,y) = 3x^2e^{xy}$, find $\frac{\partial f}{\partial x}$, f_y, f_{xx}, $\frac{\partial^2}{\partial y \partial x}f$, f_{yy}, and $f_{yyyy}(1,2)$.

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8. Let $f(x,y,z) = x^3yz^2$ and $g(x,y,z) = \ln(x^2 + y^2 + z^2)$.

   (a) Find the rates of change of f and g in the direction of $\mathbf{v} = \langle 2, -1, 2 \rangle$ at the point $(1,1,1)$.

   (b) Find the minimum and maximum rates of change of f and g at the point $(1,2,1)$, and the unit vector directions in which the minimum and maximum rates of change occur.

   (c) Find the linearizations to f and g at the point $(2,1,1)$.

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9. Consider the surface $e^{x-yz} + xz = 9$.

   (a) Find an equation for the tangent plane to the surface at $(4,2,2)$.

   (b) Assuming that z is defined implicitly as a function of x and y, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

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10. Suppose $f(x,y)$ has $\frac{\partial f}{\partial x}(1,5) = 9$, $\frac{\partial f}{\partial y}(1,5) = -3$, $\frac{\partial f}{\partial x}(2,-2) = 4$, and $\frac{\partial f}{\partial y}(2,-2) = 5$, where $x(s,t)$ and $y(s,t)$ are such that $x(1,5) = 2$, $y(1,5) = -2$, $\frac{\partial x}{\partial s}(1,5) = 3$, $\frac{\partial x}{\partial t}(1,5) = 2$, $\frac{\partial y}{\partial s}(1,5) = 4$, and $\frac{\partial y}{\partial t}(1,5) = -2$. Find:

    (a) $\frac{\partial f}{\partial s}$ at $(s,t) = (1,5)$.

    (b) $\frac{\partial f}{\partial t}$ at $(s,t) = (1,5)$.

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11. Suppose $f(x,y) = x^3 + 3xy$.

    (a) Find f_{xxy}.

    (b) Find the rate of change of f at the point $(x,y) = (1,2)$ in the direction of the origin.

    (c) Find the direction in which f is decreasing the fastest at $(x,y) = (2,0)$.

    (d) If $x = 3s^2 - 2t$ and $y = s^3t$, find $\partial f / \partial t$ when $s = 1$ and $t = 2$.

    (e) Find the linearization of f at $(x,y) = (1,3)$ and use it to estimate $f(1.01, 3.02)$.

    (f) Find an equation for the tangent plane to $z = f(x,y)$ at $(x,y) = (-1,1)$.

    (g) Find and classify the critical points for f.

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12. Find and classify the critical points for each function:

    (a) $f(x,y) = xy - x^2 - y^2 - 2x - 2y$.

    (b) $f(x,y) = x^4 + y^2 - 8x^2 + 4y$.

    (c) $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$.

    (d) $f(x,y) = x^3 - 3xy + 3y^2$.
