Note: Some answers may vary depending on the solution method.

1. (a) $\mathbf{v}+2\mathbf{w} = \langle 1, 12, 0\rangle$, $||\mathbf{v}|| = \sqrt{3^2+0^2+(-4)^2} = 5$, $||\mathbf{w}|| = \sqrt{(-1)^2+6^2+2^2} = \sqrt{41}$.
   (b) $\mathbf{v}\cdot\mathbf{w} = -11$ and $\mathbf{v}\times\mathbf{w} = \langle 24, -2, 18\rangle$.
   (c) Such a vector is $\dfrac{-\mathbf{v}}{||\mathbf{v}||} = \left\langle -\dfrac{3}{5}, 0, \dfrac{4}{5}\right\rangle$.
   (d) Such a vector is $4\dfrac{\mathbf{w}}{||\mathbf{w}||} = \left\langle \dfrac{-4}{\sqrt{41}}, \dfrac{24}{\sqrt{41}}, \dfrac{8}{\sqrt{41}}\right\rangle$.
   (e) The angle is $\theta = \cos^{-1}(\dfrac{\mathbf{v}\cdot\mathbf{w}}{||\mathbf{v}||\,||\mathbf{w}||}) = \cos^{-1}\left[\dfrac{-11}{5\sqrt{41}}\right]$.
   (f) Projection is $\text{proj}_{\mathbf{w}}\mathbf{v} = (\dfrac{\mathbf{v}\cdot\mathbf{w}}{\mathbf{w}\cdot\mathbf{w}})\mathbf{w} = \left\langle \dfrac{11}{41}, -\dfrac{66}{41}, -\dfrac{22}{41}\right\rangle$.
   (g) Area of parallelogram is $||\mathbf{v}\times\mathbf{w}|| = \sqrt{904}$.

2. Note that different forms of the answers may still be correct (e.g., if a different starting point or variation on the direction or normal vector are used).
   (a) Plane has same normal vector $\langle 1, 2, -3\rangle$ but passes through $(2, -1, 2)$. Equation is $x + 2y - 3z = -6$.
   (b) Direction vector is $\langle 3, 6, 2\rangle - \langle 2, -1, 4\rangle = \langle 1, 7, -2\rangle$. Parametrization is $\langle x, y, z\rangle = \langle 2+t,\ -1+7t,\ 4-2t\rangle$.
   (c) Line has same direction vector $\langle -2, 2, 5\rangle$ but passes through $(1, 1, 1)$. Parametrization is $\langle x, y, z\rangle = \langle 1-2t,\ 1+2t,\ 1+5t\rangle$.
   (d) Normal vector $\langle 1, 2, -3\rangle$ passing through $(0, 0, 0)$: equation is $x + 2y - 3z = 0$.
   (e) Normal vector orthogonal to $\langle 2, 1, 2\rangle - \langle 1, 0, 1\rangle = \langle 1, 1, 1\rangle$ and $\langle 3, 3, 5\rangle - \langle 1, 0, 1\rangle = \langle 2, 3, 4\rangle$, hence given by cross product $\langle 1, 1, 1\rangle \times \langle 2, 3, 4\rangle = \langle 1, -2, 1\rangle$. Equation is then $x - 2y + z = 2$.
   (f) Direction vector orthogonal to $\langle 1, 1, 2\rangle$ and $\langle 2, -1, -1\rangle$ hence given by cross product $\langle 1, 1, 2\rangle \times \langle 2, -1, 1\rangle = \langle 1, 5, -3\rangle$. Setting $z = 0$ gives $x + y = 4$ and $2x - y = 5$ yielding $x = 3$, $y = 1$: thus a point in both planes is $(3, 1, 0)$. Hence parametrization of line is $\langle x, y, z\rangle = \langle 3+t,\ 1+5t,\ -3t\rangle$.
   (g) Normal vector orthogonal to $\langle 1, 2, -1\rangle$ and $\langle 2, -1, 1\rangle$ hence given by cross product $\langle 1, 2, -1\rangle \times \langle 2, -1, 1\rangle = \langle 1, -3, -5\rangle$. Passes through $(1, -1, 2)$ hence equation is $x - 3y - 5z = -6$.

3. By the point-to-plane distance formula, the distance is $|1 + 2\cdot 3 + 2 - 5| / \sqrt{1^2+2^2+1^2} = 4/\sqrt{6}$.

4. (a) Velocity is $\mathbf{v}(t) = \mathbf{r}'(t) = \langle -3\sin(t),\ 5\cos(t),\ -4\sin(t)\rangle$.
   (b) Speed is $||\mathbf{v}(t)|| = \sqrt{9\sin^2(t) + 25\cos^2(t) + 16\sin^2(t)} = \sqrt{25} = 5$.
   (c) Arclength is $s = \int_0^1 ||\mathbf{v}(t)||\,dt = \int_0^1 5\,dt = 5$.
   (d) Acceleration is $\mathbf{a}(t) = \mathbf{r}''(t) = \langle -3\cos(t), -5\sin(t), -4\cos(t)\rangle$.
   (e) Unit tangent is $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{||\mathbf{r}'(t)||} = \left\langle -\dfrac{3}{5}\sin(t),\ \cos(t),\ -\dfrac{4}{5}\sin(t)\right\rangle$.
   (f) Unit normal is $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||} = \left\langle -\dfrac{3}{5}\cos(t),\ -\sin(t),\ -\dfrac{4}{5}\cos(t)\right\rangle$.

5. The tangent line passes through $\mathbf{r}(1) = \langle 1, 1, 1\rangle$ with direction vector $\mathbf{r}'(1) = \langle 2, 3, 4\rangle$, hence is $\langle x, y, z\rangle = \langle 1+2t, 1+3t, 1+4t\rangle$.

6. (a) By integrating and plugging in initial condition, $\mathbf{v}(t) = \langle 4,\ 8,\ 80 - 10t\rangle$.
   (b) By integrating $\mathbf{v}(t)$ and plugging in initial condition, $\mathbf{r}(t) = \langle 4t,\ 8t,\ 80t - 5t^2\rangle$.
   (c) Height is 0 when $80t - 5t^2 = 0$ so hits ground when $t = 16$s.
   (d) Desired speed is $||\mathbf{v}(16\text{s})|| = \sqrt{4^2+8^2+(-80)^2}\,\text{m/s} = \sqrt{6480}\,\text{m/s}$.

7. $\dfrac{\partial f}{\partial x} = f_x = 6xe^{xy} + 3x^2ye^{xy}$, $f_y = 3x^3e^{xy}$, $f_{xx} = 6e^{xy} + 12xye^{xy} + 3x^2y^2e^{xy}$, $\dfrac{\partial^2}{\partial y \partial x} f = f_{xy} = 9x^2e^{xy} + 3x^3e^{xy}$, $f_{yy} = 3x^4e^{xy}$, and $f_{yyyy} = 3x^6e^{xy}$ so that $f_{yyyy}(1, 2) = 3e^2$.

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8. Note that $\nabla f = \left\langle 3x^2yz^2,\, x^3z^2,\, 2x^3yz \right\rangle$ and $\nabla g = \frac{1}{x^2+y^2+z^2} \langle 2x, 2y, 2z\rangle$.

   (a) For f, we have $D_{\mathbf{v}} f = \nabla f(1, 1, 1) \cdot \frac{\mathbf{v}}{||\mathbf{v}||} = \langle 3, 1, 2\rangle \cdot \frac{1}{3} \langle 2, -1, 2\rangle = 3$.
   For g, we have $D_{\mathbf{v}} g = \nabla g(1, 1, 1) \cdot \frac{\mathbf{v}}{||\mathbf{v}||} = \frac{1}{3} \langle 2, 2, 2\rangle \cdot \frac{1}{3} \langle 2, -1, 2\rangle = 2/3$.

   (b) For f, maximum rate is $||\nabla f(1, 2, 1)|| = ||\langle 6, 1, 4\rangle|| = \sqrt{53}$ in direction of $\frac{\nabla f}{||\nabla f||} = \frac{1}{\sqrt{53}} \langle 6, 1, 4\rangle$.
   Minimum rate is $-\,||\nabla f(1, 2, 1)|| = -\sqrt{53}$ in direction of $-\frac{\nabla f}{||\nabla f||} = -\frac{1}{\sqrt{53}} \langle 6, 1, 4\rangle$.
   For g, maximum rate is $||\nabla g(1, 2, 1)|| = \left|\left|\frac{1}{6} \langle 2, 4, 2\rangle\right|\right| = \frac{1}{6}\sqrt{24}$ in direction of $\frac{\nabla g}{||\nabla g||} = \frac{1}{\sqrt{24}} \langle 2, 4, 2\rangle$.
   Minimum rate is $-\,||\nabla g(1, 2, 1)|| = -\frac{1}{6}\sqrt{24}$ in direction of $-\frac{\nabla g}{||\nabla g||} = -\frac{1}{\sqrt{24}} \langle 2, 4, 2\rangle$.

   (c) Linearization of f is $L(x, y, z) = 8 + 12(x - 2) + 8(y - 1) + 16(z - 1)$.
   Linearization of g is $L(x, y, z) = \ln(6) + \dfrac{2}{3}(x - 2) + \dfrac{1}{3}(y - 1) + \dfrac{1}{3}(z - 1)$.

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9. Surface is $f(x, y, z) = 9$ where $f(x, y, z) = e^{x-yz} + 3yz$ and $\nabla f = \langle e^{x-yz} + z,\, -ze^{x-yz},\, -ye^{x-yz} + x\rangle$.

   (a) Since $\nabla f(4, 2, 2) = \langle 3, -2, 2\rangle$, the tangent plane at $(4, 2, 2)$ is $3(x - 4) - 2(y - 2) + 2(z - 2) = 0$.

   (b) By implicit differentiation, $\dfrac{\partial z}{\partial y} = -\dfrac{f_y}{f_z} = -\dfrac{-ze^{x-yz}}{-ye^{x-yz} + x}$ and $\dfrac{\partial z}{\partial x} = -\dfrac{f_x}{f_z} = -\dfrac{e^{x-yz} + z}{-ye^{x-yz} + x}$.

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10. We need to apply the correct version of the chain rule in each case. If $s = 1$ and $t = 5$ then $x = 2$ and $y = -2$.

   (a) Here, $\dfrac{\partial f}{\partial s} = \dfrac{\partial f}{\partial x}\cdot\dfrac{\partial x}{\partial s} + \dfrac{\partial f}{\partial y}\cdot\dfrac{\partial y}{\partial s} = 4\cdot 3 + 5\cdot 4 = 32$.

   (b) Here, $\dfrac{\partial f}{\partial t} = \dfrac{\partial f}{\partial x}\cdot\dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y}\cdot\dfrac{\partial y}{\partial t} = 4\cdot 2 + 5\cdot(-2) = -2$.

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11. (a) We have $f_x = 3x^2 + 3y$, $f_{xx} = 6x$, so $f_{xxy} = 0$.

   (b) Note $\nabla f = \left\langle 3x^2 + 3y, 3x\right\rangle$ so $\nabla f(1, 2) = \langle 9, 3\rangle$. The unit vector towards the origin is $\frac{1}{\sqrt{5}} \langle -1, -2\rangle$ so the rate of change is $\langle 9, 3\rangle \cdot \frac{1}{\sqrt{5}} \langle -1, -2\rangle = -\frac{15}{\sqrt{5}} = -3\sqrt{5}$.

   (c) The direction is $-\nabla f(2, 0) = -\langle 12, 6\rangle$. As a unit vector this is $\frac{-\langle 12,6\rangle}{||-\langle 12,6\rangle||} = \frac{1}{\sqrt{180}} \langle -12, -6\rangle$.

   (d) By the chain rule, $\dfrac{\partial f}{\partial t} = \dfrac{\partial f}{\partial x} \cdot \dfrac{\partial x}{\partial t} + \dfrac{\partial f}{\partial y} \cdot \dfrac{\partial y}{\partial t}$. If $s = 1$ and $t = 2$ then $x = -1$ and $y = 2$ so evaluating everything yields $\dfrac{\partial f}{\partial t} = (3x^2 + 3y)(-2) + (3x)(s^3) = 9(-2) + (-3)(1) = -21$.

   (e) We have $L(x, y) = f(1, 3) + f_x(1, 3)(x - 1) + f_y(1, 3)(y - 3) = 10 + 12(x - 1) + 3(y - 3)$. Then $f(1.2, 2.9) \approx L(1.2, 2.9) = 10.18$.

   (f) If $x = -1$, $y = 1$ then $z = -4$. For $g(x, y, z) = f(x, y) - z = x^3 + 3xy - z$, $\nabla g = \left\langle 3x^2 + 3y, 3x, -1\right\rangle$ so $\nabla g(-1, 1, -4) = \langle 4, -3, -1\rangle$ so tangent plane is $4(x + 1) - 3(y - 1) - (z + 4) = 0$ or equivalently $4x - 3y - z = -3$.

   (g) Solving $f_x = 0$, $f_y = 0$ yields $3x^2 + 3y = 0$ and $3x = 0$ so $x = 0$ and then $y = 0$: critical point is $(0, 0)$. Then $D = f_{xx}f_{yy} - (f_{xy})^2 = (6x)(0) - 3^2 = -9$ so since $D < 0$, $(0, 0)$ is a saddle point.

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12. First we solve $f_x = f_y = 0$ to identify critical points, then use the second derivatives test ($D = f_{xx}f_{yy} - (f_{xy})^2$).

   (a) $f_x = y - 2x - 2$, $f_y = x - 2y - 2$, solving yields $(x, y) = (-2, -2)$. Then $D = (-2)(-2) - 1^2 = 3$. Yields local maximum at $(-2, -2)$.

   (b) $f_x = 4x^3 - 16x$, $f_y = 2y + 4$ so $x = -2, 0, 2$ and $y = -2$: yields $(x, y) = (-2, -2)$, $(0, -2)$, $(2, -2)$. Then $D = (12x^2 - 16)(2) - 0^2$. Yields local minima at $(-2, -2)$ and $(2, 2)$, saddle at $(0, -2)$.

   (c) $f_x = y - 1/x^2$, $f_y = x - 1/y^2$ so $y = 1/x^2$ and $x - x^4 = 0$, yielding $(x, y) = (1, 1)$ (note $x = 0$ doesn't work). Then $D = (2/x^3)(2/y^3) - 1^2$. Yields local minimum at $(1, 1)$.

   (d) $f_x = 3x^2 - 3y$, $f_y = -3x + 6y$, so $x = 2y$ and then $12y^2 - 3y = 0$ yielding $(x, y) = (0, 0)$ and $(1/2, 1/4)$. Then $D = (6x)(6) - (-3)^2$. Yields saddle at $(0, 0)$, local minimum at $(1/2, 1/4)$.
