

1. Solve the following problems:

- (a) Find the flux of  $\mathbf{F} = \langle xy^2z^2, x^2z^2, -xy^2 \rangle$  outward through the surface  $S$  made up of the portions of the six planes  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ , with  $0 \leq x, y, z \leq 1$ .
  - (b) Find the flux of  $\mathbf{F} = \langle x^3z, y^3z, 0 \rangle$  outward through the boundary of the solid region with  $x^2 + y^2 \leq 4$  and  $1 \leq z \leq 3$ .
  - (c) Find the flux of  $\mathbf{F} = \langle xy^2, yz^2, x^2z \rangle$  through the surface of the unit sphere with outward orientation.
  - (d) Find the flux of the curl  $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, d\sigma$ , where  $\mathbf{F}(x, y, z) = \langle -2y \cos(z), 2x, xe^y \rangle$  and  $S$  is the upper half of the sphere  $x^2 + y^2 + z^2 = 9$  with outward orientation.
  - (e) Find the outward flux  $\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma$  where  $S$  is the surface of the "ice cream cone" (the portion of the cone  $z = \sqrt{x^2 + y^2}$  that lies inside the sphere  $x^2 + y^2 + z^2 = 1$  along with that portion of the sphere that lies above the cone), and  $\mathbf{F}(x, y, z) = \langle x + 2y, x + 2z, y + 2z \rangle$ .
  - (f) Find the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $\mathbf{F}(x, y, z) = \langle x + y^2, y + z^2, z + x^2 \rangle$  and  $C$  is the quadrilateral with vertices  $(0, 0, 1), (1, 0, 0), (1, 1, -1)$ , and  $(0, 1, 0)$  lying in the plane  $x + y + z = 1$ , oriented in the counterclockwise direction when viewed from above.
  - (g) Find the circulation of the vector field  $\mathbf{F} = \langle 2xy, x^2, y \rangle$  around the counterclockwise boundary of the portion of the surface  $z = x^2y$  that lies above the plane region with  $0 \leq x \leq 1$  and  $0 \leq y \leq x$ .
  - (h) Find the flux of  $\mathbf{F} = \langle 3x^2y, xy^2, 8xy \rangle$  outward through the surface  $S$  made up of the portions of the five planes  $x = 0, x = 1, y = 0, y = 1, z = 1$ , with  $0 \leq x, y, z \leq 1$ .
  - (i) Find the flux of  $\mathbf{F} = \langle xy^2 + e^z, x^2y + e^{2z}, \sqrt{x^2 + y^2} \rangle$  through the portion of the surface  $z = 1 - x^2 - y^2$  that lies above the  $xy$ -plane.
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