Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	5	5	10	10	10	10	10	10	10	10	10	100
Score:												

PRINT Your name here: ____

PRINT Your instructor's name here: ____

Please check that you have 11 different pages.
Answers from your calculator, without supporting work, are worth zero points.

1. (5 points) Find a standard equation of the tangent plane at the point $(8, 2, 1)$ to the level surface of the function $f(x, y, z) = x - y^2 + z^2$.

Answer:

2. (5 points) Suppose that the xy-plane is occupied by a heated metal plate of temperature $T = T(x,y)$, measured in Celsius, at the point (x,y), where x and y meters, and

$$\frac{\partial T}{\partial x}\Big|_{(1,2)} = -1°\text{C/m} \quad \text{and} \quad \frac{\partial T}{\partial y}\Big|_{(1,2)} = 2°\text{C/m}.$$

A path given by the parametrization $\vec{r}(t) = \left(t, \frac{4}{1+t^2}\right)$ is traced on the plate, where t is in seconds. What is the instantaneous rate of change of the temperature along the path at the point $(1,2)$?

Answer:

3. (10 points) A bug is crawling on the surface of a hot plate in which the temperature at the point x units to the right and y units up is given by

$$T(x, y) = 3xe^y + 2y \ln x + y.$$

(a) If the bug is at the point (1, 0), in what direction should it move to cool off the fastest? What is the rate at which temperature drops in this direction?

Answer:

(b) If the bug is at the point (1, 3), what is the rate of change of the temperature with respect to distance if the bug is moving to the Southeast? Note that a vector pointing directly southeast is (1, -1).

Answer:

4. (10 points) Find all four critical points of

$$f(x, y) = 2x^3 + 9xy^2 + 15x^2 + 27y^2$$

and classify each one as a point where f has a local maximum value, a local minimum value, or a saddle point.

Critical Point	Write Local Max, Local Min or Saddle

5. (10 points) Find the global maximum of the function

$$f(x,y) = 3x^2 + xy + 2y^2$$

over the filled-in triangle with vertices

$$(-1,0), \ (1,0) \ \text{ and } \ (0,2).$$

Answer:

6. (10 points) Find the volume of the solid below the surface $z = 4 - x^2 - y^2$ and above the xy-plane.

Answer:

7. (10 points) Find the mass of the unit ball (i.e. a sphere of radius 1 meter) centered at the origin, with density $\delta(x, y, z) = z^2$ kg/m^3.

Answer:

8. Consider the vector field

$$\mathbf{F}(x, y) = (P(x, y), Q(x, y)) = (6x^2 + 2x \sin y, x^2 \cos y + 4y^3).$$

(a) (2 points) Find the 2-dimensional curl of $\mathbf{F}$, i.e. $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$.

Answer:

(b) (8 points) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the above vector field $\mathbf{F}$ along the curve C in $\mathbb{R}^2$ parameterized by

$$\mathbf{r}(t) = (2t \cos t, 2t \sin t),$$

where t ranges in the interval $0 \leq t \leq 2\pi$.

Answer:

9. (10 points) Let C be the circle of radius 3 meters, centered at the origin and oriented counterclockwise. Consider the force field

$$\mathbf{F}(x, y) = \left(11x + 7y,\, 2x + e^{-y^4}\right) \text{ Newtons where } x \text{ and } y \text{ are in meters.}$$

Calculate the work done by $\mathbf{F}$ on an object that moves around the curve C.

Answer:

10. (10 points) Find the flux of the vector field

$$\mathbf{F} = (3x + 1, 2xe^z, 3y^2 z + z^3)$$

across the outward oriented faces of a cube **without the front face at** $x = 2$ and with vertices at (0,0,0), (2,0,0), (0,2,0) and (0,0,2).

Answer:

11. (10 points) Let

$$\mathbf{F} = (3x^2z - 2y, 3x + 3y^2z^2, 5xe^z + y^2).$$

(a) Find curl $\mathbf{F}$.

Answer:

(b) Evaluate $\iint_M (\vec{\nabla} \times \mathbf{F}) \cdot \mathbf{n}\, dS$ where M is the part of the sphere $x^2 + y^2 + z^2 = 25$ below $z = -4$ oriented upwards.

Answer: