Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													
out of	8	8	8	8	8	8	10	8	8	10	8	8	100

Math 2321 Final Exam

December 14, 2015

Instructor's name____________________________ Your name____________________________

Please check that you have 9 different pages.

Answers from your calculator, without supporting work, are worth zero points.

1) Consider the function $f(x, y) = 4x^4y^4 - \sqrt{x}\, e^{y-1}$.

a) (6 points) What is the linearization $L(x, y)$ of $f(x, y)$ at $\mathbf{a} = (1, 1)$?

ANSWER: ____

b) (2 points) Use your linearization to estimate $f(1.01, 0.99)$. (The "exact" answer without using linearization is worth 0 points.)

ANSWER: ____

2) (8 points) Suppose that the real-valued function $f = f(x, y, z)$ is differentiable at $(-3, \ln 5, 1)$, and that $\nabla f(-3, \ln 5, 1) = (-3, 5, 2)$. Suppose that

$$x(t) = -3e^t, \quad y(t) = \ln(t^2 + 5), \quad z(t) = \cos t.$$

Calculate df/dt at $t = 0$.

ANSWER: ____

3) (4 points each) The elevation above sea level, in meters, of a saddle between two mountains is given by $z = xy + 1000$, where x and y are also in meters. Suppose that a goat is at a point near the saddle where $(x, y) = (10, -20)$.

a) In what direction (as a unit vector in the xy-plane) should the goat head to ascend as rapidly as possible?

ANSWER: ____

b) At what rate, in units of meters of elevation per meter of distance in the xy-plane, will the goat ascend if it moves in the direction from (a)?

ANSWER: ____

4) (8 points) Find an equation for the tangent plane to the level surface of the function $f(x, y, z) = x^2y - \sin z$ at the point $\mathbf{p} = (2, 1, 0)$.

ANSWER: ____

5) (8 points) Consider the surface parameterized by $\mathbf{r}(u,v) = \left(u - v, u + v, \dfrac{u^2 - v^2}{2}\right)$. Describe, in parametric form, the tangent plane of $\mathbf{r}$ at $(u,v) = (4,2)$.

ANSWER: ________________________________

6) (8 points) Find and classify the critical points of the function $f(x,y) = x^3 - 6xy + 8y^3$ as points where f attains a local maximum value, a local minimum value, or has a saddle point.

ANSWER: ________________________________

7) (10 points) Find the global maximum and global minimum values of $f(x, y) = x^2 + 2y^2 - 4y$ subject to the constraint $x^2 + y^2 = 9$.

ANSWER: ____

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8) (8 points) Evaluate the iterated integral $\displaystyle\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2+x^3}\, dx\, dy.$

ANSWER: ____

9) (8 points) An ice cream shop sells ice cream at a price of 2 dollars per pound. For one serving of ice cream, the machine fills the cone, which has a shape given by $z = \sqrt{3x^2 + 3y^2}$, and then continues adding ice cream up to the surface of the sphere given by $x^2 + y^2 + z^2 = 25$, where x, y, and z are in inches. Suppose that the ice cream they serve has a density of 0.04 lb/in^3. How much will it cost to get one serving of ice cream?

ANSWER: ________________________________

10) Suppose $\mathbf{F}(x, y) = (y^3 e^x - x, 3y^2 e^x + 4x)$ represents a force field in Newtons, where x, y, and z are in meters.

a) (3 points) Compute the 2-dimensional curl of $\mathbf{F}$.

ANSWER: ______________________________

b) (7 points) Find the work done by $\mathbf{F}$ on a particle which moves along the top half of the circle of radius 1, centered at the origin, from $(1, 0)$ to $(-1, 0)$.

ANSWER: ______________________________

11) (8 points) Calculate the total outward flux of the vector field $\mathbf{F}(x, y, z) = (x^3, y^3, 1)$ across the boundary of the solid bounded by the paraboloid where $z = 1 - x^2 - y^2$ and the xy-plane.

ANSWER: ____

12) Let $\mathbf{F}(x, y, z) = (x^2 - z, x + y^2, z^2)$. Let C be the curve of intersection of the plane $z = 1 - x - y$ and the cylinder $x^2 + y^2 = 9$. Assume that C is oriented counterclockwise, as viewed from above. Let M be the region in the plane $z = 1 - x - y$ which is inside the curve C, so that C is the boundary of M.

a) (2 points) Calculate the curl of $\mathbf{F}(x, y, z)$.

ANSWER: ________________________________

b) (2 points) Determine a unit normal to M which is compatible with the given orientation on its boundary C.

ANSWER: ________________________________

c) (4 points) Use Stokes' Theorem to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

ANSWER: ________________________________