Definition 1 An integer x is even if $x = 2k$ for some integer.

Definition 2 An integer x is odd if $x = 2k + 1$ for some integer k.

Definition 3 An integer a is divisible by an integer b or b divides a, denoted $b|a$, if there is an integer c such that $bc = a$.

Definition 4 An integer p is prime if $p > 1$ and the only positive divisors of p are 1 and p.

Definition 5 An integer is composite if there is an integer b such that $b|a$ and $1 < b < a$.

Definition 6 Set A is a subset of set B $(A \subseteq B)$ if every element of A is also an element of B.

Definition 7 Two sets A and B are equal if $A \subseteq B$ and $B \subseteq A$.

Definition 8 The intersection of sets A and B is $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

Definition 9 The union of sets A and B is $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Definition 10 Let A be a set. The power set of A, denoted 2^A, is the set of all subsets of A.

Definition 11 The difference of sets A and $A - B = \{x : x \in A \ and \ x \notin B\}$.

Definition 12 The symmetric difference of sets A and B is $A \,\Delta\, B = (A - B) \cup (B - A)$.

Definition 13 The Cartesian product of sets A and B is $A \times B = \{(a, b) : a \in A, b \in B\}$.

Definition 14 R is a relation on a set A if $R \subseteq A \times A$. _Notation_: $(x, y) \in R$ is equivalent to xRy.

Definition 15 The inverse of relation R is $R^{-1} = \{(x, y) : (y, x) \in R\}$

Definition 16 Let R be a relation on set A.

- R is reflexive if xRx for all $x \in A$.
- R is irreflexive if $x \not R x$ for all $x \in A$.
- R is symmetric if $xRy \to yRx$ for all $x, y \in A$.
- R is antisymmetric if $(xRy \wedge yRx) \to x = y$ for all $x, y \in A$.
- R is transitive if $(xRy \wedge yRz) \to xRz$ for all $x, y, z \in A$.

Definition 17 A relation R on A is an equivalence relation if R is reflexive, symmetric, and transitive.

Definition 18 Let n be a positive integer. Then the "congruence modulo n" relation on $\mathbb{Z}$ is defined as follows: $x \equiv y \ (mod\ n)$ if $n|(x - y)$.

Definition 19 Let R be an equivalence relation on a set A and let $a \in A$. The equivalence class of a is

$$[a] = \{x \in A : xRa\}$$

Definition 20 $n! = n(n-1)\cdots(3)(2)(1), \ 0! = 1$

Definition 21 $(n)_k = n(n-1)\cdots(n-k+1)$

$$(n)_k = \frac{n!}{(n-k)!}$$

Definition 22 $\binom{n}{k}$ = the number of k-element subsets of an n-element set

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Definition 23 Let f be a relation from set A to set B. Then f is a **function** from A to B, $f: A \to B$, if

- the set of all possible first elements of f, called the **domain** of f, is A
- $(x, y) \in f$ and $(x, z) \in f$ imply $y = z$.

Notation: $(x, y) \in f$ is equivalent to $y = f(x)$.

The **image** of f is the set im $f = \{y \in B: (x, y) \text{ for some } x \in A\}$.

Definition 24 Let A and B be sets, and $f: A \to B$. Then

- f is **one-to-one** if $f(x) = f(y)$ implies $x = y$.
- f is **onto** if for each $b \in B$, there exists an $a \in A$ such that $f(a) = b$.
- f is a **bijection** if it is one-to-one and onto.

Definition 25 Let A, B, and C be sets, and $f : A \to B$ and $g : B \to C$. Then $g \circ f$ is a function from A to C and $(g \circ f)(a) = g\big(f(a)\big)$.

Basic Proposition An integer is either odd or even but not both.