1. The volume of a cylindrical block of ice is $V = \pi r^2 h$. If the radius r is currently 10cm and decreasing at 1cm/min and the height h is currently 20cm and decreasing at 3cm/min, how fast is the volume decreasing?

2. Sand falls into a conical pile whose height is always 5/2 its radius. If the height of the sandpile is currently 5 meters and sand is being deposited onto the pile at a rate of π cubic meters per minute, how fast are the height and radius of the pile increasing? (Note: The volume of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

3. John is sailing south towards Moya Island and Aeryn is sailing west away from Moya Island. Currently, John is 120km away from the island and approaching it at 10kph, and Aeryn is 50km away from the island and moving away at 50kph. How fast is the distance between John and Aeryn changing?

4. Find the absolute minimum and maximum of f on the given interval, and all places where they occur:
   (a) $f(x) = x^3 - 6x^2 + 9x$ on $[0, 4]$.
   (b) $f(x) = x + 16/x$ on $[1, 8]$.
   (c) $f(x) = x^2 e^x$ on $[-3, 1]$.
   (d) $f(x) = 2x + 4\sin(x)$ on $[0, \pi]$.
   (e) $f(x) = \sqrt{x}(x^2 - 5)$ on $[0, 3]$.
   (f) $f(x) = x/(1 + x^2)$ on $[-2, 2]$.

5. Use the Intermediate Value Theorem + Rolle's Theorem to show $f(x) = x^3 + 3x + 1$ has exactly 1 real root.

6. For each function, find and classify all critical numbers as minima/maxima/neither, find all inflection numbers, and find all intervals where f is increasing, decreasing, concave up, and concave down:
   (a) $f(x) = x^2 - 4x + 5$.
   (b) $f(x) = x^3 + 9x^2 - 21x + 2$.
   (c) $f(x) = x^5 + 5x^4 + 7$.
   (d) $f(x) = x^4 - 2x^2 + 3$.
   (e) $f(x) = 2x + 4\sin(x)$ on $(0, 2\pi)$.
   (f) $f(x) = xe^{-x^2/8}$.

7. Find the following limits or explain why they do not exist (note ∞ and $-\infty$ are possible values):
   (a) $\lim_{x\to 0} \dfrac{e^x - 1}{\sin(3x)}$.
   (b) $\lim_{x\to 4} \dfrac{x^2 - 16}{\sqrt{x} + 2}$.
   (c) $\lim_{x\to 0} \dfrac{\sin(x^2)}{1 - \cos(x)}$.
   (d) $\lim_{x\to 0} \left[\dfrac{2}{\sin(2x)} - \dfrac{1}{\tan(x)}\right]$.
   (e) $\lim_{x\to\infty} \dfrac{x^3 + 2x^2 + 5}{2x^2 - x + 1}$.
   (f) $\lim_{x\to 2+} (x^2 - 4)\ln(x - 2)$.
   (g) $\lim_{x\to\infty} (1 + 2/x)^{3x}$.
   (h) $\lim_{x\to\infty} (x^2 + 3x)^{4/\ln(x)}$.

8. The sum of three positive numbers is 12 and two of them are equal. Find the largest possible product.

9. Allan has 12 feet of string. He uses some to form a square and the rest to form a 3-4-5 right triangle. Find the maximum and minimum possible total areas of Allan's two shapes.

10. You cut squares of side length s from each of the four corners of a rectangular piece of paper measuring 14in by 30in, and fold the resulting shape up into a box with no top. What value of s maximizes the volume?

11. Find $f(x)$ if $f''(x) = 12x^2 - e^x + \sin(x)$, where $f'(0) = 1$ and $f(0) = 2$.

12. A population of goats grows at a rate proportional to its current size. In 2010 the population is 500 and in 2020 the population is 1500. Find the population in 2030.

13. Find the left-endpoint, midpoint, and right-endpoint Riemann sums for each function:
    (a) $f(x) = \sqrt{x}$ on $[0, 4]$ with 2 equal subintervals.
    (b) $f(x) = x^3$ on $[0, 1]$ with 5 equal subintervals.

14. Evaluate the following integrals:
    (a) $\int_0^4 (x^3 + x + \sqrt{x})\, dx$.
    (b) $\int \left(\dfrac{1}{\sqrt{1 - x^2}} + \dfrac{1}{1 + x^2}\right) dx$.
    (c) $\int_0^{\pi/2} (4\sin(t) - \cos(t))\, dt$.
    (d) $\int_2^2 \sqrt{3e^{2x} + \sin(x)}\, dx$.
    (e) $\int_0^{\pi/4} \sec(x)\tan(x)\, dx$.
    (f) $\int_1^e \dfrac{x^2 - x + 1}{x}\, dx$.
    (g) $\int_0^{\pi/3} \sec^2(\star)\, d\star$.
    (h) $\int_2^4 dx$.
    (i) $\int (x^2 + 1)^2\, dx$.
    (j) $\int_{\pi/6}^{\pi/3} \dfrac{\sin(2x)}{\cos^2(x)}\, dx$.
    (k) $\int_1^3 (3^q - 2^q)\, dq$.
    (l) $\int \sqrt{\sin^2 x + \cos^2 x}\, dx$.

15. Suppose that $\int_1^3 f(x)\, dx = 4$, $\int_3^4 f(x)\, dx = 5$, $\int_0^1 g(x)\, dx = 1$, and $\int_0^4 g(x)\, dx = -2$. Find:
    (a) $\int_1^4 f(x)\, dx$.
    (b) $\int_1^4 g(x)\, dx$.
    (c) $\int_1^4 [f(x) - g(x)]\, dx$.
    (d) $\int_1^3 [2f(x) + x]\, dx$.